Electrical Measurement & Instrumentation

DIPLOMA IN ELECTRICAL ENGINEERING, 4th SEM

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MEASUREMENTS

BASICS :

$A_t = 10A$ (True value)

$A_m = 9.8A$. (Measured value).

$$\boxed{\text{Static error } (\delta A) = A_m - A_t}$$

$$= 9.8 - 10 = (-) 0.2 A$$

Static correction (δC) = $-\delta A$

$$= -(-0.2) = 0.2A.$$

Correction is to be done for A_m to get A_t.

$2A \pm 1A$ $\rightarrow$ 50% error } not good $1000A \pm 10A$ $\rightarrow$ 1% error } good.

$$\boxed{\text{Relative static error} = \frac{A_m - A_t}{A_t}}$$

Accuracy :

It is a measure of closeness with which an instrument reading approaches the true value of the quantity being measured (Mesurand).

precission :

Reproducability of measurements.

Measurand : quantity under measurement.

Sensitivity :-

It is the ratio of magnitude of o/p signal to the mag. of i/p signal under measurement.

Dead time :-

It is the time required by a measurement system to respond to change in the measurement. dead time depends on damping factor selected for meter. range : 0.6 to 0.8. ($\xi < 1$ under damped with ξ_1 0.6 to 0.8)

Dead zone :-

It is the largest change of i/p quantity for which there is no o/p of the instr.

Resolution or Descrimination :-

The smallest increment in i/p which can be detected with certainity by an instr. is known as Resolution.

$R_1 = 1000\,\Omega \pm 10\,\Omega$ 990   1010

$R_2 = 500\,\Omega \pm 5\,\Omega$ 495   505

(990+495)   (1010+505)

$R_1 + R_2 = 1500 \pm 15$ 1485   1515

$R_1 - R_2 = 500 \pm 15$ 485   515

(990−505)   (1010−495)

* Resultant error in addition & subtraction of quantity can be obtained by adding all individual errors. They should be expressed in absolute values.

$V = 230V \pm 2\%$

$I = 10A \pm 1\%$

$P = 2300 \pm 3\%$ $(V \cdot I)$

$R = 23 \pm 3\%$ $\left(\frac{V}{I}\right)$

* Resultant error in product & division of quantities can be obtained by adding all individual errors and they should be expressed in percentage values.

$I = 10\,A \pm 2\%$

$R = 500\,\Omega \pm 3\%$

Then power $P = I^2 R$

$= 50000 \pm [2\times 2\% + 1\times 3\%]$

$= 50000 \pm 7\%$

Resultant error in polynomials can be obtained by the above method.

NOTE:

It is preferrable to measure the quantity instead of calculation. In the calculation of quantity the error will be more.

eg: $V \Rightarrow 2\%$

By meters
$I \Rightarrow 1\%$
$P \Rightarrow 3\%$
$\rightarrow Pf \Rightarrow 2\%$

By calculation,

$Pf = \frac{P}{VI} = 6\%$

Types of Errors :-

(a). Gross Errors :-

This class of errors mainly comes by human mistakes in reading, recording and calculating the measurements.

(b). Systematic Errors :

(1). Instrumental Errors :-

→ Due to inherent short comings { Errors in instr. }

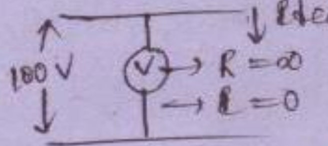
→ Misuse of instr.

1.2 A

0 – 5 A (I) ✓

0 – 100 A (II)

→ Loading effects



	Ideal	but practical
	$R = \infty$	R = finite
	$I = 0$	I = finite

(2). Environmental Errors :-

Due to temp, electro magnetic effects.

(3). Observational Errors :-

↳ parallox errors

(C). Random Errors :-

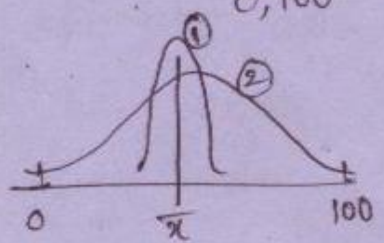
→ These errors occurs due to unknown source or cumulative of different sources together.

→ Random errors can be compensated by statistical Analysis.

↳ Mean $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

Eg : 51, 49 → $\bar{x} = 50$

0, 100 → $\bar{x} = 50$ ← not advisable.



$d_1 = x_1 - \bar{x}$

$d_2 = x_2 - \bar{x}$

$\vdots \qquad \vdots$

$d_n = x_n - \bar{x}$.

Avg. deviation $= \dfrac{|d_1| + |d_2| + \cdots + |d_n|}{n}$.

(σ) Std. deviation $= \sqrt{\dfrac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}}$

(σ^2) Variance $= (S.D)^2$

True value : of the quantity to be measured may be ~~~~ defined as an avg. of infinite no. of measured values when the avg. deviation due to various contributing factors tends to be zero.

1.1). $A_m = 6.7\,A$

$A_t = 6.54\,A$

Error $= A_m - A_t$

(δA) $= 0.16\,A$

correction factor $\delta C = -\delta A$

$= -0.16\,A$.

1.2). Range : $0 - 2.5V$

$A_t = 1.5V$

$A_m = 1.46V$

Error $\delta A = A_m - A_t$

$= -0.04V$

correction $\delta C = -\delta A$

$= 0.04V$

% Error $= \frac{-0.04}{1.5} \times 100$

$= -2.67\%$

% Error $= \frac{-0.04}{2.5} \times 100$ (w.r.t. Full scale).

$= -1.6\%$

1.3). $R = 5000 \pm 10\%$

$= (5000 \pm 500)\,\Omega$

$\rightarrow R = 4500\,\Omega$ to $5500\,\Omega$.

1.4). $(0-10)A \longrightarrow 1.5\%$

Error $= 10 \times \frac{1.5}{100}$

$= 0.15\,A$

$A_m = 2.5\ A$

$I = 2.5 \pm 0.15 \Rightarrow 2.35$ to $2.65\ A$

Relative error $= \dfrac{0.15}{2.5} \times 100$

$= 6\%$

1.5). Diameter $= 100$ mm

$= 0.1\text{m} \pm 1\%$

velocity $= 1\ \text{m/s} \pm 3\%$

flow rate = Area × velocity

$= \dfrac{\pi D^2}{4} \times v$

$= \dfrac{\pi}{4}(0.1)^2 \times 1$

$= 0.785 \times 10^{\cancel{-4}\,-2}\ m^3/s$

$= 7.85 \times 10^{\cancel{-5}\,-3}\ m^3/s. \pm [2 \times 1\% + 1 \times 3\%]$

$= 7.85 \times 10^{\cancel{-5}\,-3} \pm 5\%$

1.6). $R_4 = \dfrac{R_1 R_2}{R_3}$.

$R_1 = 500\ \Omega \pm 1\%$. $R_2 = 615 \pm 1\%$ $R_3 = 100 \pm 0.5\%$

(a). $R_4 = \dfrac{500 \times 615}{100} = 3075\ \Omega$

(b). $R_4 = 3075 \pm (1 + 1 + 0.5)\%$

$= 3075 \pm 2.5\%$

$= 3075 \pm 76.88\ \Omega$.

1.7). $I/P = 6250\ \omega \pm 2\%$

$= 6250 \pm 125\ \omega$

$O/P = 5000\omega \pm 3\% = 5000 \pm 150\ \omega$

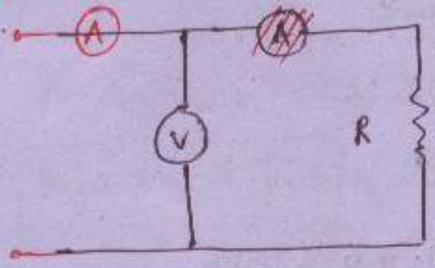
$$\text{Losses} = i/p - o/p$$

$$= (1250 \pm 275)\,W$$

$$= 1250\,W \pm 22\%$$

$$\eta = \frac{o/p}{i/p} = \frac{5000}{6250} = 0.8 \pm 5\%$$

18).



$S_v = 1000\,\Omega/V$

$(0-150)V$ scale

$I = 5\,mA$

$= 5 \times 10^{-3}\,A$.

(a). Apparent resistance $= \frac{V}{I}$

$$= \frac{100}{5\times10^{-3}} = 20\,k\Omega.$$

(b). Resistance of voltmeter

$$R_V = S_V \times 150$$

$$= 1000 \times 150 = 150\,k\Omega.$$

(c). $\frac{1}{R_T} = \frac{1}{R_V} + \frac{1}{R}$

$$\Rightarrow \frac{1}{20} = \frac{1}{150} + \frac{1}{R}$$

$$\Rightarrow R = \cancel{23\,k\Omega}.\ 23.077\,k\Omega$$

$$\%\ \text{error} = \frac{A_m - A_t}{A_t}$$

$$= \frac{20-23}{23} \times 100$$

$$= \cancel{-11.8\%}.\ -13.33\%.$$

1.9). $V = 123.4V$ Range : $(0-250)V \rightarrow 1\%$

$I = 283.5\ mA$ Range : $(0-500)\ mA \rightarrow 1\%$

Error $= 250 \times \frac{1}{100} = 2.5V$

Error $= 500 \times \frac{1}{100} = 5\ mA$

$V = 123.4 \pm 2.5\ V$

$= 123.4V \pm 2.02\ \%$

$I = 283.5 \pm 5\ mA$

$= 283.5\ mA \pm 1.76\ \%$

$$R = \frac{V}{I} = \frac{123.4}{283.5m} = 435.2\ \Omega$$

$R = 435.2 \pm (2.02 + 1.76)\%$

$= 435.2 \pm 3.78\ \%$

1.10). $A_t = 2.5\ V$

$A_m = 2.46\ V$

Error $= A_m - A_t = -0.04$

% Error (True value) $= \frac{-0.04}{2.5} \times 100$

$= -1.6\ \%$

% Error (F.S.D) $= \frac{-0.04}{4} \times 100$

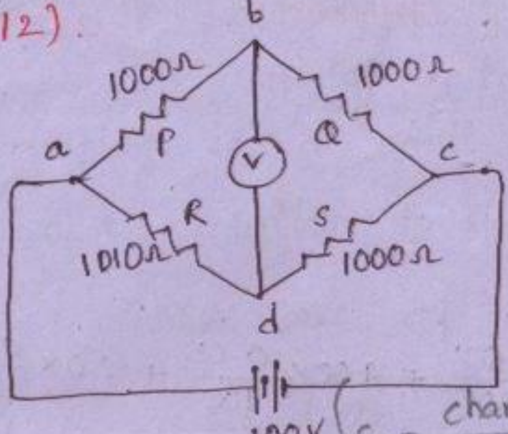
$= -1\ \%$

1.11). Resolution $= \frac{100}{400} \times \frac{1}{5}$

$= 0.05V$

$= 50\ mV$

1.12).



$V_{bd} = V_b - V_d$

$= 50 - \left[100 - \frac{100}{1000+1010} \times 1010\right]$

change in o/p quantity

$= 0.248$

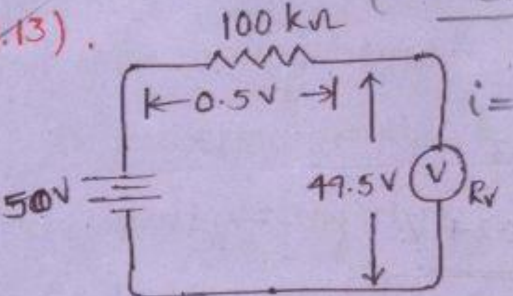
Bridge sensitivity

$S_B = \frac{\text{change in volt.}}{\text{change in Resistance}}$ ← change in i/p quantity

$S_B = \frac{0.248}{10} = 0.0248\ V/\Omega$

$= 24.8\ mV/\Omega$

1.13).



$i = \frac{0.5}{100} = \frac{49.5}{R_V}$

$\Rightarrow R_V = 9900\ k\Omega$

$= 9.9\ M\Omega.$

1.14).

$\text{Resolution} = \frac{50}{100} \times \frac{1}{5}$

$= 0.1\ kN/m^2$

100 div ⟶ $50\ kN/m^2$

1/5 div ⟶ ?

1.15). (0 – 10) A

$\text{Error} = 10 \times \frac{1.5}{100} = 0.15$

(a). $I = 1.5\ A$

$\%\ \text{Error} = \frac{0.15}{1.5} \times 100$

$= 10\%$

(b). $I = 5A$

$\%\ \text{Error} = \frac{0.15}{5} \times 100 = 3\%$

(c). $I = 2.5\ A$

$\%\ \text{Error} = \frac{0.15}{2.5} \times 100$

$= 6\%$

1.16). units : $\pm 0.2\%$

Hundreds : $\pm 0.05\%$

Tens : $\pm 0.1\%$

Thousands : $\pm 0.02\%$

$$R = 3425\,\Omega.$$

$$\text{Error} = 3000 \times \frac{0.02}{100} + 400 \times \frac{0.05}{100} + 20 \times \frac{0.1}{100} + 5 \times \frac{0.2}{100}$$

$$= 0.83\,\Omega$$

$$\therefore R = 3425 \pm 0.83$$

$$= 3425 \pm 0.024\%$$

1.17). $P \Rightarrow 2\%$

$I \Rightarrow 1\%$

$V \Rightarrow 1\%$

$$pf = \frac{\cancel{V}P}{VI}$$

$$\Rightarrow (2+1+1)\%$$

$$\Rightarrow 4\%$$

Absolute Instruments:

These instr.s gives the mag. of the quantity in terms of physical constraints of the instr.

Ex:- Tangent Galvonometer.

Rayleigh's current balance.

Secondary Instruments:

These instr.s use the measurand value directly by the o/p indicated by the instr.

Ex:- Ammeter, voltmeter, wattmeter etc.

Analog Instruments:

An analog instr. in which the o/p or display is a conti. fun. of time and having const. relation to its i/p.

These are of 3 types.

(1). Indicating type:

Ex:- Ammeter, voltmeter, pf meter, wattmeter

(2). Recording type:

Ex:- Sesmograph, ECG, recording voltmeter etc.

(3). Integrating type:

Ex:- Energymeter $\rightarrow \int p\,dt$ = Energy $\Rightarrow$ KWH

$1\ KWH = 1000 \times 3600 = 3.6 \times 10^{6}$ J/s

charge meter $\rightarrow \int i\,dt$ = charge $\Rightarrow$ 1 coulomb

1 coulomb = 1 Ampere - sec $\qquad \int v\,dt$ = flux

1 Amp hour = 3600 coulombs.

Odo meter $\rightarrow \int N\,dt$ = distance

Total flow meter $\rightarrow \int$ flow rate. dt = Total flow

Indicating Type Instruments:-

The following 3 torques are responsible for the operation of an indicating instr.

(1). Deflecting torque:

It is responsible for the required deflection of pointer or moving system. for a given value of the measurand.

It will be produce by any of the effects caused by flow of current through ckts.

$\Longrightarrow$ At this position, $(T_d = T_c)$

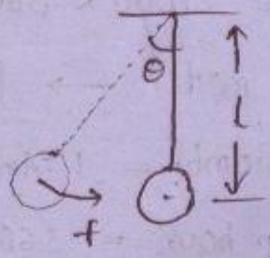
<u>control torque</u> :

It is responsible for the control of the movement of pointer and make definite deflection for a given value of measurand.

It is useful to bring back moving system to zero position once the measurand is removed.

There are 2 types of getting the control torque.

(i). <u>Gravity control</u> :-



control torque = wl sinθ

$T_c \alpha \sin\theta$ The control torque is proportional to sin of deflection angle.

The gravity control instr. should always operate in vertical position.

(ii). <u>spring control</u>:

hair springs made up of silicon branze, hard rolled silver, platinum silver etc are used for getting the control torque.

$\boxed{T_c \propto \theta}$ (deflection angle)
(control torque)

Damping torque :

It is responsible to supress the oscillations of the pointer at the final deflected position

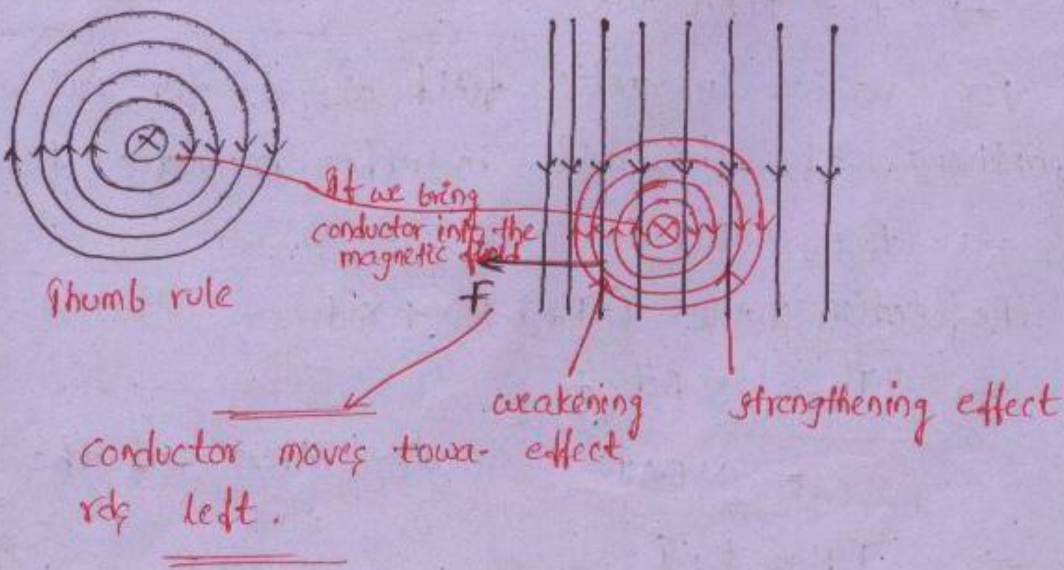
various types of damping torques are :

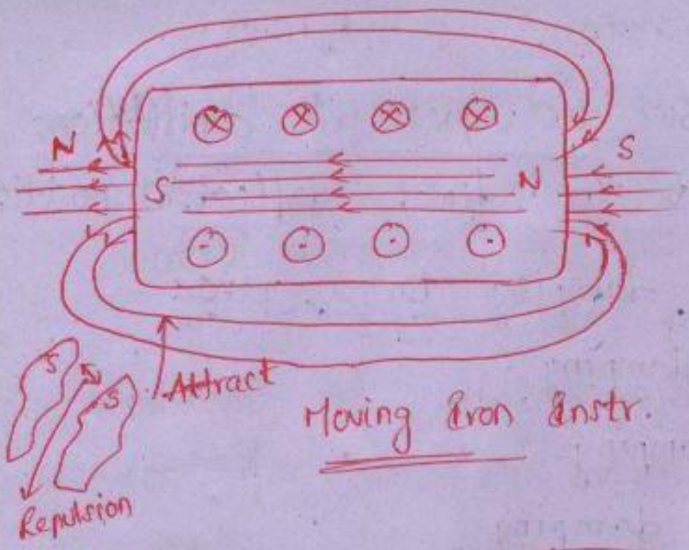
(i). Eddy current damping
(ii). Air friction damping
(iii). fluid friction damping
(iv). Electromagnetic damping

⟹ The following meters are used for the measurement of voltage & current.

(1). PMMC ⟶ DC
(2). MI ⟶ AC & DC
(3). dynamometer instr.
(4). Electro static instr.
(5). Thermal instr.
(6). Rectifier type instr.
(7). Induction type instr. ⟶ Only AC

(3)–(6): Both AC & DC

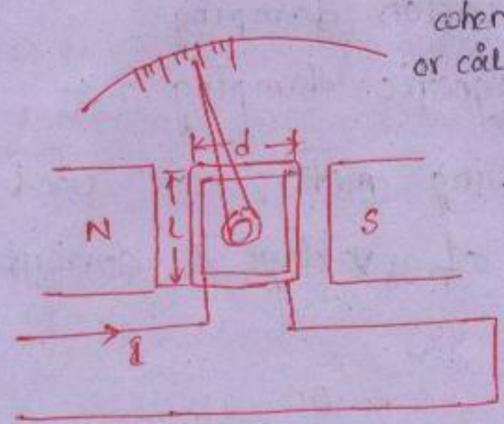




Moving Iron Instr.

PMMC :-

when a current carrying condu. or coil



As per field theory mech. force on a conductor can be $\hat{f} = \hat{B} \times \hat{il}$

$\Rightarrow f = BIL \sin\alpha \; ; \; \alpha = \angle B, il$

Let the coil consists of 'N' no. of turns, force on each coil side is expressed as

$\Rightarrow f = NBil \sin\alpha$

for radial magnetic field $\alpha = 90$, for all positions of the coil in the magnetic field

$\Rightarrow f = NBil$

Deflection torque $(T_d) = f \times d$

$\Rightarrow \boxed{T_d = NBil \cdot d}$

$= NBAI$.

A - Area of the coil.

$\Rightarrow T_d = K_d I$

K_d - deflection const.

$K_d = NBA$ units : Nm / Amp

Springs are used for getting the control torque.

$$T_c \propto \theta \Rightarrow T_c = K_c \theta$$

$K_c \rightarrow$ spring const. units : Nm / rad or Nm/deg

At final deflected position,

$$T_d = T_c$$

$$\Rightarrow \boxed{NBAI = K_c \theta} \rightarrow \text{PMMC.}$$

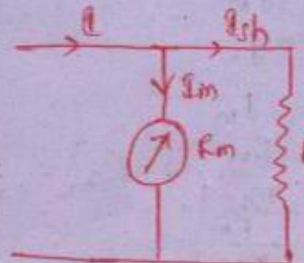
$$\Rightarrow \boxed{\theta \propto I.} \rightarrow \text{Scale is uniform \& linear.}$$

$\Rightarrow$ Eddy current damping is employed in PMMC.
For this purpose an Al metalic frame is used.
In the case of Ammeter, coil carries the current to be measured. In the case of voltmeter a current proportional to volt. pass through the coil. By measuring this current volt. can be evaluated.

The basic PMMC meter current carrying capacity max. of 1A. This is due to presence of spring in the physical part of the current from external ckt to moving coil.

Ammeter Shunts :-

These are the small resistances connected across the moving coil [basic meter], to increase the current measuring capacity.



$$I_m R_m = I_{sh} \cdot R_{sh}$$

$$= (I - I_m) R_{sh}$$

$$\Rightarrow I_m [R_m + R_{sh}] = I R_{sh}$$

$$\Rightarrow \frac{I}{I_m} = m = \frac{R_m + R_{sh}}{R_{sh}}$$

$$\Rightarrow m = \frac{R_m}{R_{sh}} + 1.$$

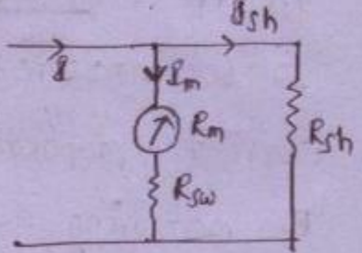
$$\Rightarrow m - 1 = \frac{R_m}{R_{sh}}$$

$$\Rightarrow \boxed{R_{sh} = \frac{R_m}{m-1}}$$; m – multiplying power $= \frac{I}{I_m}$ $(m > 1)$.

* shunt resistance is always less than the meter resistance.

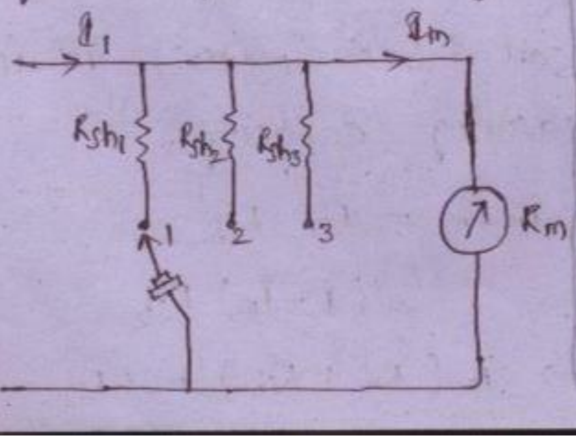
* Manganin is the material used for the preparation of dc shunts. since it possess negligible temp. coe. of resistance.

* swamping resistance will be connected in series with the meter and is prepared with same material as that of shunt. It is suitable for minimizing the errors due to temp. variations.



Multi range Ammeters :-

In this independent shunts are employed for independent ranges.



$m_1 = \frac{I_1}{I_m}$

$m_2 = \frac{I_2}{I_m}$

$m_3 = \frac{I_3}{I_m}$

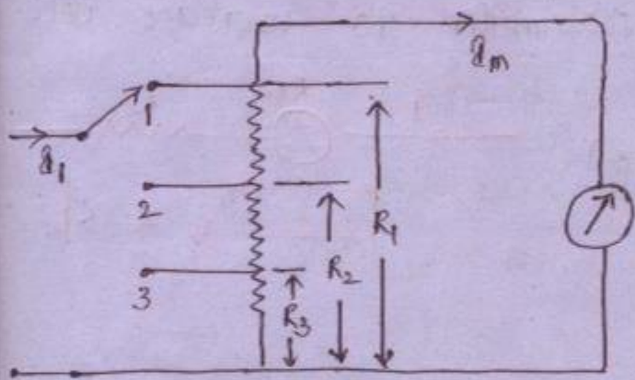
$R_{sh_1} = \frac{R_m}{m_1 - 1}$; $R_{sh_2} = \frac{R_m}{m_2 - 1}$

$R_{sh_3} = \frac{R_m}{m_3 - 1}$.

The switch to be employed in this meter is "make before break".

There is a possibility of damaging the meter due to passage of heavy currents during the transition from one range to another range.

Universal shunt of Ar



$m_1 = \frac{I_1}{I_m}$; $m_2 = \frac{I_2}{I_m}$

$m_3 = \frac{I_3}{I_m}$.

Let switch is at pos. 1.

$I_m R_m = (I_1 - I_m) R_1$

$\Rightarrow I_m [R_m + R_1] = I_1 R_1$

$\Rightarrow \frac{I_1}{I_m} = m_1 = \frac{R_m + R_1}{R_1}$

$\Rightarrow m_1 = \frac{R_m}{R_1} + 1$

$\Rightarrow m_1 - 1 = \frac{R_m}{R_1}$

$\Rightarrow \boxed{R_1 = \frac{R_m}{m_1 - 1}}$

Let switch is at pos. 2:-

$(I_2 - I_m) R_2 = I_m (R_m + R_1 - R_2)$

$\Rightarrow I_2 R_2 = I_m (R_m + R_1 - R_2 + R_2)$

$\Rightarrow \frac{I_2}{I_m} = \frac{R_m + R_1}{R_2} = m_2$

$\Rightarrow m_2 = \frac{R_1 + R_m}{R_2}$

$\Rightarrow \boxed{R_2 = \frac{R_m + R_1}{m_2}}$

Let switch is at pos. 3 :-

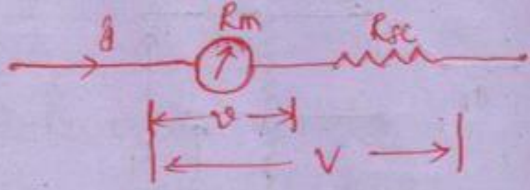
$(I_3 - I_m) R_3 = I_m (R_m + R_1 - R_3)$

$\Rightarrow \frac{I_3}{I_m} = m_3 = \frac{R_m + R_1}{R_3}$

$\Rightarrow \boxed{R_3 = \frac{R_m + R_1}{m_3}}$.

Voltage Multipliers :-

These are the high resistances connected in series with basic meter to increase volt. measuring capacity.



$\boxed{m = \frac{V}{v_m}} = \frac{I(R_m + R_{se})}{I R_m}$

$\Rightarrow m = 1 + \frac{R_{se}}{R_m}$

$\Rightarrow \frac{R_{se}}{R_m} = m - 1 \Rightarrow \boxed{R_{se} = R_m (m-1)}.$ $m > 1$

* Series resistance is always more than the meter resistance.

MOVING IRON :-

whenever an iron piece is placed in the visinity of MF produced by a current carrying coil then it will be subjected to mech. force. MI meter operation is based on change in self inductance of coil. Deflection torque expression can be obtained by conservation of energy principle electrical energy supplied = change in stored energy + mech. work. done

$\Rightarrow$ Electrical energy supplied $= V I \, dt$.

$= \left(L \cdot \frac{dI}{dt} + I \cdot \frac{dL}{dt}\right) I \, dt$

$= L I \, dI + I^2 dL.$

stored energy $= \frac{1}{2} L I^2$.

change in stored energy $= d\left[\frac{1}{2} L I^2\right]$

$$= \frac{1}{2} I^2 dL + \frac{1}{2} L \cdot 2I \cdot dI.$$

$$= \frac{1}{2} I^2 dL + L I \, dI$$

mech. work done $= T_d \cdot d\theta$

$$\Rightarrow L I dI + I^2 dL = \frac{1}{2} I^2 dL + L I dI + T_d \cdot d\theta$$

$$\Rightarrow \frac{1}{2} I^2 dL = T_d \cdot d\theta$$

$$\Rightarrow T_d = \frac{1}{2} I^2 \cdot \frac{dL}{d\theta}$$

Springs are used for getting the control torque. So $T_c \propto \theta$

$$\Rightarrow T_c = k_c \theta.$$

At final deflected position

$$T_d = T_c$$

$$\Rightarrow \boxed{\frac{1}{2} I^2 \underbrace{\frac{dL}{d\theta}}_{\text{Henry/rad.}} = k_c \theta.} \rightarrow \text{MI.}$$

where $\frac{dL}{d\theta}$ is change in self inductance of coil w.r.t deflection angle.

$\boxed{\theta \propto I^2} \rightarrow$ Scale is non-linear & it varies in square fashion.

It is assumed that $\frac{dL}{d\theta}$ is const.

* Suitable for both AC & DC measurements.
* Air friction damping is employed for MI meter.

→ Linear scale with the MI Meters :-

$\theta \propto I$ $\quad\quad \frac{1}{2} I^2 \cdot \frac{dL}{d\theta} = k_c \theta$

$\Rightarrow I \propto \theta$

$\Rightarrow I = k_1 \theta \quad \Rightarrow \frac{1}{2} (k_1 \theta)^2 \cdot \frac{dL}{d\theta} = k_c \cdot \theta$

$$\Rightarrow \frac{1}{2} \cdot \theta \cdot \frac{dL}{d\theta} = \frac{k_c}{k_i^2}$$

$$\Rightarrow \theta \cdot \frac{dL}{d\theta} = \frac{2k_c}{k_1^2} = \text{const.}$$

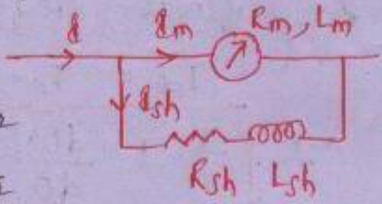
$$\Rightarrow \theta \cdot \frac{dL}{d\theta} = \text{A constant}$$

* To obtain a linear scale with MI meters, $\frac{dL}{d\theta}$ should be variable and has to vary while satisfying the condi $\boxed{\theta \cdot \frac{dL}{d\theta} = \text{const.}}$

* Current measuring capacity of MI, is more than PMMC. [may a max. of 20 A].

* To increase the current measuring capacity shunts may be employed.

* Constantan is employed for preparation of AC shunts:



$$\frac{I_m}{I_{sh}} = \frac{Z_{sh}}{Z_m} = \frac{\sqrt{R_{sh}^2 + (\omega L_{sh})^2}}{\sqrt{R_m^2 + (\omega L_m)^2}}$$

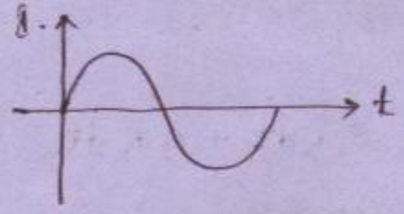
$$= \frac{R_{sh}\sqrt{1 + \left(\omega \frac{L_{sh}}{R_{sh}}\right)^2}}{R_m \sqrt{1 + \left(\frac{\omega L_m}{R_m}\right)^2}}$$

Time const. of shunt   Time const. of meter.

* If $\boxed{\frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}} \Rightarrow \frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}$

The errors due to freq. variation can be eliminated by making the time const.s of meter & shunt should be equal.

$T_d = NBA \cdot I$. If PMMC subjected to AC:



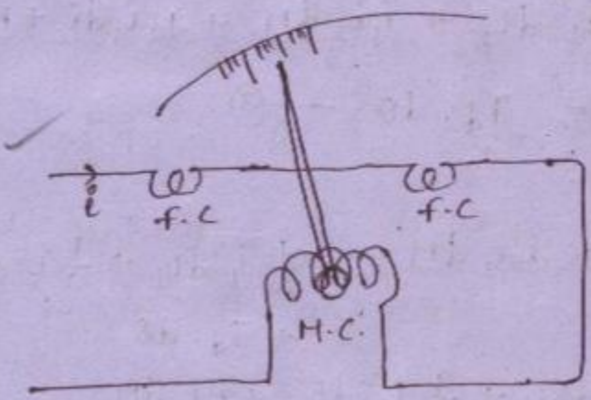
If PMMC is subjected to AC quantity, the following may occur.

(1). pointer will be oscillating around zero position if the freq. is low.

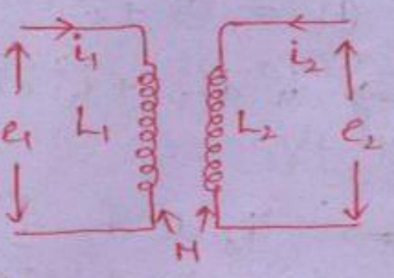
(2). pointer is at stand still at zero pos. if the freq. is high.

(3). To get unidirc. torque in moving coil instr. it is required to replace permanent magnets by electromagnets.

ELECTRO DYNAMO METER TYPE INSTRUMENTS:



FC → fixed coil
MC → Moving coil



operating principle is based on change in M b/w the two coils.

* In the case of ammeter & voltmeter both the coils are connect in series and carries the same current.

Let i_1, i_2 be the instantaneous currents passing through FC & MC res.

Acc. to Energy conservation principle

Electrical energy supplied = change in stored energy + mech. work done.

$\psi_1 = L_1 i_1 + M i_2$

$\psi_2 = L_2 i_2 + M i_1$

Electrical energy supplied $= e_1 i_1 dt + e_2 i_2 dt$

$= \frac{d\psi_1}{dt} i_1 . dt + \frac{d\psi_2}{dt} . i_2 . dt$

$= i_1 d\psi_1 + i_2 d\psi_2$

$= i_1 d[L_1 i_1 + M i_2] + i_2 d[L_2 i_2 + M i_1]$

$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + M i_1 di_2 + L_2 i_2 di_2$

$+ i_2^2 dL_2 + i_1 i_2 dM + M i_2 di_1$

Stored energy $= \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$

change in stored energy $= L_1 i_1 di_1 + \frac{1}{2} i_1^2 dL_1$

$+ L_2 i_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM + M i_1 di_2 + M i_2 di_2$

mech. work done $= T_d . d\theta$ — ③

① = ② + ③

$\Rightarrow i_1^2 dL_1 + i_2^2 dL_2 + i_1 i_2 dM = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + T_d . d\theta$

$\Rightarrow T_d . d\theta = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$

The change in self inductances are neglected compare to mutual inductance

$\rightarrow T_d . d\theta = i_1 i_2 dM$

$\Rightarrow \boxed{T_d = i_1 i_2 . \frac{dM}{d\theta}}$

DC :

$i_1 = I_1$; $i_2 = I_2$

$T_d = I_1 I_2 . \frac{dM}{d\theta}$

AC:

Let $i_1 = I_{m_1} \sin \omega t$

$i_2 = I_{m_2} \sin(\omega t - \alpha)$

$$T_{d\,(\text{instantaneous})} = i_1 i_2 \frac{dM}{d\theta}$$

$$= I_{m_1} \sin \omega t \cdot I_{m_2} \sin(\omega t - \alpha) \cdot \frac{dM}{d\theta}.$$

Avg. deflection torque $(T_d) = \frac{1}{2\pi}\int_0^{2\pi}\left(i_1 i_2 \frac{dM}{d\theta}\right) d\omega t$

$$T_d = \frac{1}{2\pi}\int_0^{2\pi} I_{m_1} I_{m_2} \cdot \sin \omega t \cdot \sin(\omega t - \alpha) \cdot \frac{dM}{d\theta} \cdot d\omega t$$

$$= \frac{I_{m_1} I_{m_2}}{4\pi}\int_0^{2\pi} \cos\alpha - \cos(2\omega t - \alpha) \cdot d\omega t \cdot \frac{dM}{d\theta}$$

$$= \frac{I_{m_1} \cdot I_{m_2}}{4\pi} \cdot \frac{dM}{d\theta}\left[\cos\alpha \cdot \omega t - \frac{1}{2}\sin(2\omega t - \alpha)\right]_0^{2\pi}$$

$$= \frac{I_{m_1}}{\sqrt{2}} \cdot \frac{I_{m_2}}{\sqrt{2}} \cdot \frac{1}{2\pi} \cdot \frac{dM}{d\theta}\left[\cos\alpha \cdot 2\pi\right]$$

$$\Rightarrow \boxed{T_d = I_1 \cdot I_2 \cdot \cos\alpha \cdot \frac{dM}{d\theta}}$$

Here I_1, I_2 be the rms values of the currents passing through coils and having an angle b/w I_1 & I_2 is α.

DC AMMETER:

$I_1 = I_2 = I$.

$$T_d = I^2 \cdot \frac{dM}{d\theta}.$$

AC AMMETER:

$I_1 = I_2 = I$ & $\alpha = 0$.

$$T_d = I^2 \cdot \frac{dM}{d\theta}.$$

springs are used for control purpose. so

$T_c = k_c \theta$.

At final deflection position,

$T_d = T_c$

$$\Rightarrow \boxed{I^2 \cdot \frac{dM}{d\theta} = k_c \cdot \theta} \rightarrow \text{Dynamometer}$$

$\frac{dM}{d\theta}$ is, the change in M b/w two coils w.r.t. deflection angle. units Henry/rad.

* $\boxed{\theta \propto I^2}$

Scale is non-linear and varies in square (x^2) fashion

* Air friction damping is employed for the dynamo meter type instr.

→ STANDARD FOR DC VOLT:

STANDARD CELL.

→ STANDARD FOR AC VOLT: Rayleigh's current balance

In the calibration of AC meters transfer type instr.s are used. Dynamo meter type is useful as transfer instr. at power freq. range, since it possess good accuracy for both DC & AC.

At high freq. thermal instr. are used as transfer instr.

ELECTRO STATIC TYPE INSTRUMENTS:-

operating principle is based on change of capacetance. $C = \frac{\epsilon A}{d}$.

capacitance of a shunt capacetor can be varied either by overlaping area b/w the plates or distance b/w the plates.

* These are best suitable for measurement of high voltages.

deflection torque can be obtained by the principle of conservation of energy Law.

Electrical energy Supplied = change in stored energy + mech. work done.

Electrical energy supplied = $V I\,dt$

$$= v\left[c \cdot \frac{dv}{dt} + v \cdot \frac{dc}{dt}\right] dt$$

$$= cv \cdot dv + v^2 dc \quad \text{—} (1)$$

stored energy $= \frac{1}{2} cv^2$.

change in stored energy $= d\left[\frac{1}{2} cv^2\right]$

$$= cv \cdot dv + \frac{1}{2} v^2 \cdot dc \quad \text{—} (2)$$

linear motion :

$$cv dv + v^2 dc = cv \cdot dv + \frac{1}{2} v^2 dc + f dx$$

$$\Rightarrow \boxed{f = \frac{1}{2} v^2 \cdot \frac{dc}{dx}}$$

Angular motion :

$$cv dv + v^2 dc = cv \cdot dv + \frac{1}{2} v^2 dc + T_d \cdot d\theta$$

$$\Rightarrow \boxed{T_d = \frac{1}{2} v^2 \cdot \frac{dc}{d\theta}}$$

spring control is used, so $T_c \propto \theta$.

At final deflection position,

$$T_d = T_c.$$

$$\Rightarrow \boxed{\frac{1}{2} v^2 \frac{dc}{d\theta} = k_c \cdot \theta} \Rightarrow \boxed{\theta \propto v^2}$$

$\frac{dc}{d\theta}$ is change in capacitance w.r.t. deflection and unit : farad/rad.

→ There are 2 types of electrostatic meters,

(1). Quadrant Electrometer:-

suitable for measurement of high voltages.

(2). Kelvin multi cellular :-

sutable for measurement of low voltages.

THERMAL TYPE :

(1). Suitable for high freq. applications.

(2). capable to measure current which is of non-sinusoidal nature also.

(3). These instr.s employs heating effect the flow of current through the ckt.

3 Types :

(1). Hot wire instr.s

(2). Bolo meter.

(3). Thermo couple instr.s

HOT WIRE :

In this meter expansion of metal caused by flow of current through sensitive element. This element is prepared with metal having more value of coe. of expansion.

* The sensitive element is prepared by platinum irradium alloy.

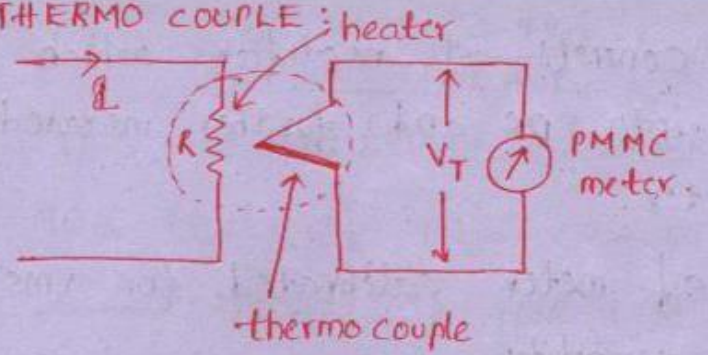
BOLO METER :

In this instr. change in Resis. caused by heating effect of current employed for the evaluation of the current.

If the sensitive element with +ve temp coe. then its resistance increases with increment of temp.

If element having -ve temp. coe., its resistance decreases with increment in temp.

THERMO COUPLE :



(1). It is a joint of two dissimilar junctions. As the temp. of the junction increases it generates a volt. known as thermal emf. This effect known as seabeck effect.

(2). deflection of PMMC meter $\theta \propto V_T$.

$$\propto a\Delta T + b(\Delta T)^2 + \ldots$$

$$\rightarrow \theta \propto a(I^2R) + \ldots$$

$$\Rightarrow \boxed{\theta \propto I^2}$$

deflection of PMMC is proportional to I^2, hence it is suitable for AC & DC measurement.

(3). Thermal instr-s are suitable as transfer instr-s for high freq. operation.

* Some of the combination of metals for making thermo couple

(i). Iron - constantan
(ii). platinum - Chromium
(iii). chromel - alumen

RECTIFIER TYPE :

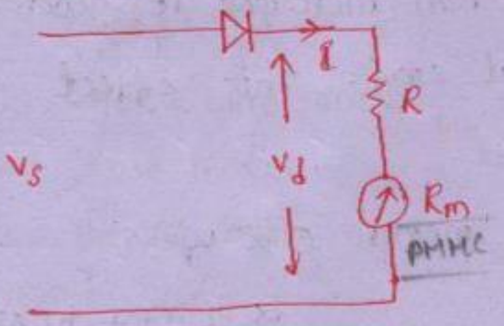
(1). Suitable for electronic measurements ie lower magnitude of volt-s.

(2). 2 types:

(a). Amplifier - rectifier type
(b). rectifier - Amplifier type.

* The meter consists of rectifier which converts AC into DC and further measured by PMMC meter.
* The reading of meter calibrated for rms value of AC quantity.

HALF WAVE RECTIFIER TYPE :



DC :

If meter is fed with dc volt. of V_s then

$V_d = V_s$

$I = \frac{V_d}{R+R_m} = \frac{V_s}{R+R_m}$.

AC :

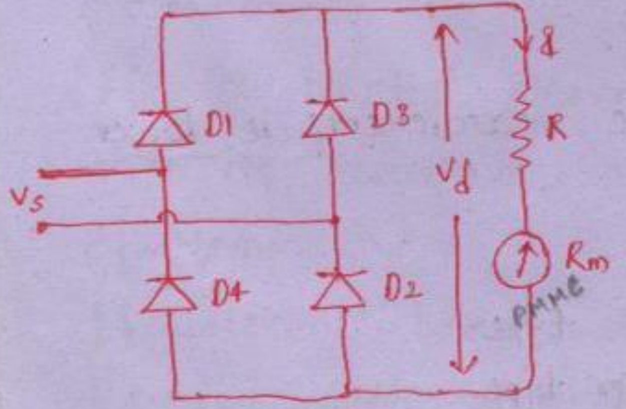
$V_d = \frac{V_m}{\pi} = \frac{\sqrt{2}V_s}{\pi} = 0.45\ V_s$.

$I = \frac{V_d}{R+R_m} = \frac{0.45\ V_s}{R+R_m}$.

If the meter is fed with AC volt. V_s which is same that of dc volt. Then,

* The sensitivity for AC of HWR type instr. is 45% that of DC.

FULL WAVE RECTIFIER TYPE :



DC :

$V_d = V_s$

$I = \frac{V_d}{R+R_m} = \frac{V_s}{R+R_m}$

AC :

$V_d = \frac{2V_m}{\pi} = \frac{2\sqrt{2}}{\pi}V_s = 0.9V_s$

$$I = \frac{V_d}{R + R_m} = \frac{0.9 V_s}{R + R_m}$$

* The AC sensitivity of FWR type instr. is 90% that of DC.
* AC sensitivity of FWR type instr. is 2 times that of HWR type instr.
* The DC sensitivity of FWR type instr. is same as that of HWR instr.
* Form factor $= \frac{RMS}{Average}$.
* Deflection of PMMC is proportional to avg. value
* ∴ Scale of the meter is calibrated for rms value by multiplying avg. value with FF. Usually it will be taken as 1.11

for sinusoidal wave $FF = 1.11$.

for square wave $FF = 1$.

2.1)

$N = 100$

$d = 20$ mm

$l = 30$ mm

$B = 0.1T$

$I = 10$ mA

$K_c = 2 \times 10^{-6}$ N-m/deg.

$T_d = NBA \cdot I$

$= 100 \times 0.1 \times 30 \times 20 \times 10^{-6} \times 10 \times 10^{-3}$

$= 60$ μNm

$NBA \cdot I = K_c \theta \longrightarrow$ Moving coil

$\Rightarrow 60 \times 10^{-6} = 2 \times 10^{-6} \times \theta \Rightarrow \theta = 30°$.

2.2). $R = 10000\ \Omega$

$A = 30 \times 30\ mm^2$

$N = 100$

$B = 0.08\ wb/m^2$

$k_c = 3 \times 10^{-6}\ Nm/deg.$

$V = 200V.$

$NBA \cdot I = k_c \theta$

$\Rightarrow 100 \times 0.08 \times 30 \times 30 \times 10^{-6} \times \left(\frac{200}{10000}\right) = 3 \times 10^{-6} \cdot \theta$

$\Rightarrow \theta = 48^\circ.$

2.3).

$R_m = 1\ \Omega$

$V = 250V$

$R_{se} = 4999\ \Omega$

$I_m = \frac{250}{4999+1} = 0.05A.$

(a). $R_{sh} = \frac{1}{499}\ \Omega$; $\frac{I}{I_m} = 500$

$R_{sh} = \frac{R_m}{m-1}$

$I = 500 \times 0.05$

$= 25A.$

$\Rightarrow \frac{1}{499} = \frac{1}{m-1}$

$\Rightarrow m = 500.$

(b). $I = 50A$

$m = \frac{I}{I_m} = \frac{50}{0.05} = 1000$

$R_{sh} = \frac{R_m}{m-1}$

$= \frac{1}{1000-1} = \frac{1}{999}\ \Omega$

2.4). $A = 40 \times 30 \ mm^2$

$N = 100$

$T_c = 0.25 \times 10^{-3} \ Nm$

50 divisions

$B = 1T$

1 V / div

$R_V = 10000 \ \Omega$

$V = 50 V$

$I \cdot NBA = k_c \theta$

$\longrightarrow I = 2.083 \times 10^{-3} A$

1 div — 1V

50 div — ? = 50V.

$\therefore R_{Total} = \frac{50}{2.083 \times 10^{-3}} = 24000 \Omega$

$NBA \cdot I = k_c \theta$

$\Rightarrow 100 \times 1 \times \frac{50}{R_T} = 0.25 \times 10^{-3}$

$\Rightarrow R_T = 24000 \ \Omega$

$R_{se} = 24000 - 10000$

$= 14000 \Omega.$

2.5). $I_m = 10 mA$

$V_m = 100 mV$

$R_m = \frac{V_m}{I_m} = 10 \ \Omega$

(a). $m = \frac{I}{I_m} = \frac{100}{10 \times 10^{-3}}$

$= 10000$

$R_{sh} = \frac{R_m}{m-1} = \frac{10}{9999} \Omega$

$= 0.001 \Omega$

(b). $m = \frac{V}{V_m}$

$= \frac{1000}{100 \times 10^{-3}}$

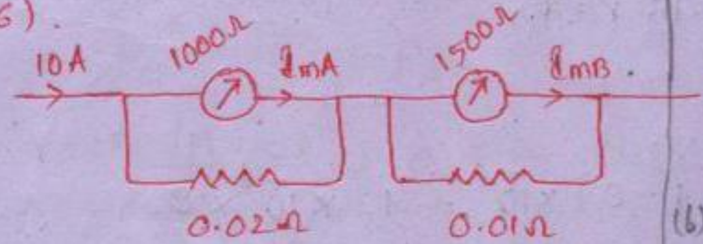
$= 10000$

$R_{se} = R_m (m-1)$

$= 10 (9999)$

$= 99.99 \ k\Omega$

2.6).



$I_{mA} = 10 \times \frac{0.02}{1000 + 0.02}$

$= 0.2 \ mA$

$I_{mB} = 10 \times \frac{0.01}{1500 + 0.01}$

$= 0.0667 \ mA.$

If shunts are interchanged, then

$I'_{mA} = 10 \times \frac{0.01}{1000 + 0.01} = 0.1 mA \Rightarrow \left(\frac{0.2}{2} mA\right)$

$I'_{mB} = 10 \times \frac{0.02}{1500 + 0.02} = 0.133 mA \longrightarrow (0.0667 \times 2)$

Ammeter A shows only 5 A b'coz current through the meter is half

Ammeter B shows 20 A since the current is doubled.

2.7).

$$L = (8 + 4\theta - \frac{1}{2}\theta^2)\ \mu H$$

$$k_c = 12 \times 10^{-6}\ Nm/rad.$$

$$\frac{1}{2} I^2 \cdot \frac{dL}{d\theta} = k_c \cdot \theta \longrightarrow \text{moving iron}$$

$$\Rightarrow \frac{dL}{d\theta} = (4 - \theta)\ \mu H/rad.$$

(a). $I = 1A$

$$\frac{1}{2}(1)^2 \cdot [4 - \theta] \times 10^{-6} = 12 \times 10^{-6} \times \theta$$

$$\Rightarrow \theta = 0.16 \text{ rad}$$

(b). $I = 2A$

$$\Rightarrow \frac{1}{2}(2)^2 [4 - \theta] \times 10^{-6} = 12 \times 10^{-6} \times \theta$$

$$\Rightarrow \theta = 0.57 \text{ rad}$$

(c). $I = 3A$

$$\frac{1}{2}(3)^2 [4 - \theta] \times 10^{-6} = 12 \times 10^{-6} \cdot \theta$$

$$\Rightarrow \theta = 1.16 \text{ rad}$$

(d). $I = 5A$

$$\frac{1}{2}(5)^2 [4 - \theta] \times 10^{-6} = 12 \times 10^{-6} \times \theta$$

$$\rightarrow \theta = 2.04 \text{ rad}$$

2.8). $L = (0.01 + C\theta)^2 \ \mu H$

$I_1 = 1.5 \ A \ ; \ \theta_1 = 90^\circ$

$I_2 = 2 \ A \ ; \ \theta_2 = 120^\circ.$

$$\frac{1}{2} I^2 \cdot \frac{dL}{d\theta} = k_c \cdot \theta$$

$$\frac{dL}{d\theta} = 2C[0.01 + C\theta] \ \mu H/\text{rad}.$$

$$\Rightarrow \frac{\frac{1}{2}(1.5)^2[2C(0.01 + C\overset{90^\circ}{\theta})] = k_c \times 90^\circ}{\frac{1}{2}(2)^2[2C(0.01 + 120^\circ C)] = k_c \times 120^\circ}$$

$$\Rightarrow C = -47.6 \times 10^{-6}$$

2.9).

$I = 25 \ A$

$$\frac{dM}{d\theta} = 0.0035 \ \mu H/\deg. = \frac{0.035}{\pi/180} \cdot \mu H/\text{rad}$$

$k_c = 10^{-6} \ Nm/\deg.$

$$I^2 \cdot \frac{dM}{d\theta} = k_c \cdot \theta \longleftarrow \text{Electro dynamo meter.}$$

$$\Rightarrow (25)^2 \cdot 0.0035 \times 10^{-6} \times \frac{180}{\pi} = 10^{-6} \times \theta$$

$$\Rightarrow \theta = 125^\circ.$$

2.10).

$I = 10A$

$k_c = 0.1 \times 10^{-6} \ Nm/\deg$

$\theta = 110^\circ$

$$I^2 \frac{dM}{d\theta} = k_c \cdot \theta$$

$$\Rightarrow (10)^2 \cdot \frac{dM}{d\theta} = 0.1 \times 10^{-6} \times 110$$

$$\Rightarrow \frac{dM}{d\theta} = 0.11 \ \mu H/\text{rad}.$$

$$M = 2\,\mu H + \left[0.11\times10^{-6}\times\left[110\times\frac{\pi}{180}\right]\right]$$

1.91

$$\Rightarrow M = 2.21\ \mu H.$$

$$\frac{dM}{d\theta} = \frac{M_2 \sim M_1}{\theta_2\sim\theta_1} \Rightarrow M_2 = M_1 + \frac{dM}{d\theta}(\theta_2\sim\theta_1)$$

2.11). $V_m = 100\,V$

HWR : $R = 10\,\Omega$

(a). Hot wire : $V_{or} = \frac{V_m}{2} = \frac{100}{2} = 50V$ — measures rms values

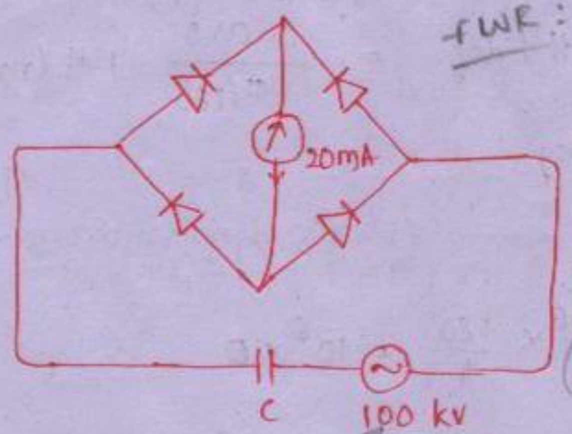
$I = \frac{50}{10} = 5A.$

(b). moving coil — measures avg. values

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi}$$

$$I = \frac{100}{\pi\times10} = 3.183\,A. \quad \left(V_{rms} = \frac{V_m}{\sqrt2}\right)$$

2.12).



FWR : $V_{dc} = \frac{2V_m}{\pi} = \frac{2\sqrt2 V_s}{\pi}$

$$= \frac{2\sqrt2\times100\times10^3}{\pi}$$

$$= 90.03\ kV$$

$$20\times10^{-3} = \frac{90.03\times10^3}{X_c} \quad \left(I_o = \frac{V_o}{X_c}\right)$$

$$\Rightarrow X_c = 4.5\times10^6\,\Omega$$

$$\Rightarrow C = \frac{1}{\omega X_c}$$

$$= \frac{1}{2\pi\times50\times4.5\times10^6}$$

$$= 707\ pF.$$

$$= 707\times10^{-12}\ f$$

2.14).

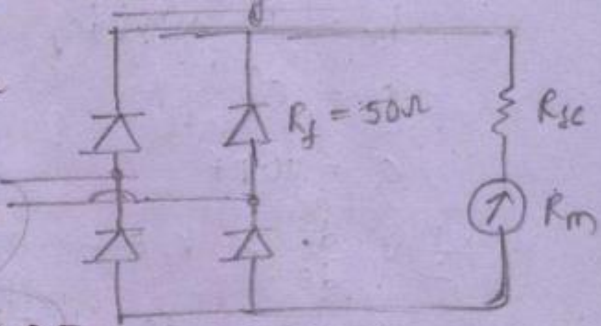
$R_m = 250\,\Omega$

$I_m = 1\,mA$

$R_D = 50\,\Omega$

(rms) $V = 25\,V$

At a time 2 diodes are conducting. so



$$V_{dc} = \frac{2V_m}{\pi}$$

$$= \frac{2\sqrt{2} \times 25}{\pi} = 22.5\,V$$

$$I_m = \frac{V_{dc}}{R_T} \qquad (R_T = 2R_f + R_{se} + R_m)$$

$$\Rightarrow R_T = \frac{22.5}{1 \times 10^{-3}} = 22.5\,k\Omega$$

$$R_{se} = 22500 - (250 + 2 \times 50)$$

$$= 22.15\,k\Omega$$

$$\Omega/V = \frac{22500}{25}$$

$$= 900\,\Omega/V.$$

2.15).

Electro static

diameter = 8 cm

$x = 4\,mm$

$f = 0.002\,N$

$$f = \frac{1}{2} V^2 \cdot \frac{dc}{dx}$$

$$C = \frac{\epsilon A}{x}$$

$$\frac{dc}{dx} = -\frac{\epsilon A}{x^2} \qquad A = \frac{\pi d^2}{4}$$

$$\Rightarrow 0.002 = \frac{1}{2} V^2 \left[\frac{-8.854 \times 10^{-12} \times 1 \times \pi \left(\frac{0.08^2}{4}\right)}{(4 \times 10^{-3})^2} \right]$$

$$\Rightarrow V = \cancel{11.99\,V}\ 1199\,V$$

2.16) $V = 1000\ V$

$k_c = 10^{-7}\ Nm/deg$

$\theta = 80^\circ$

$C = 10pf$

$\frac{1}{2} V^2 \frac{dc}{d\theta} = k_c \theta \longrightarrow$ Electro static

$\Rightarrow \frac{1}{2}(1000)^2 \cdot \frac{dc}{d\theta} = 10^{-7} \times 80$

$\Rightarrow \frac{dc}{d\theta} = 0.16\ pf/rad$

$\Rightarrow C = 10 + (.16 \times 80 \times \frac{\pi}{180})$

$= 32.34\ pf.$

2.17).

$V = 3000\ V$

$k_c = 7.6 \times 10^{-6}\ Nm/rad$

$\theta = 80^\circ = 80 \times \frac{\pi}{180} \cdot rad. = 1.39\ rad$

$\frac{1}{2} V^2 \cdot \frac{dc}{d\theta} = k_c \cdot \theta$

$\rightarrow \frac{1}{2}(3000)^2 \cdot \frac{dc}{d\theta} = 7.6 \times 10^{-6} \times 80 \times \frac{\pi}{180}$

$\Rightarrow \frac{dc}{d\theta} = 2.358 \times 10^{-12}\ f/rad.$

$\Rightarrow dc = 2.358 \times 10^{-12} \times 1.39$

$= 3.29\ pf$

2.18).

$\theta \propto I^2$

$I_1 = 10A$

$\theta_1 = \theta$

$I_2 = ?$

$\theta_2 = \frac{\theta}{2}$

$\frac{\theta_1}{\theta_2} = \frac{I_1^2}{I_2^2} \Rightarrow I_2 = \sqrt{50} = 7.07A$

MEASUREMENT OF RESISTANCE :

RESISTANCES :

(1). Low resistance : $<1\,\Omega$, below $1\,\Omega$.

(2). Medium : ~~1kΩ/1000Ω~~ $1\,\Omega$ to $100\,k\Omega$

(3). High : greater than $100\,k\Omega$

Medium resistance is represented by a two terminal resistive element.

* Medium resistance can be measured by,

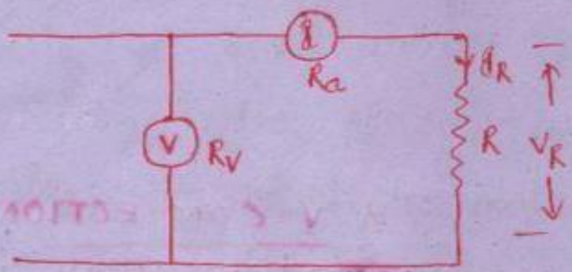
(1). Ammeter - voltmeter method

(2). Substitution method

(3). Wheatstone bridge

(4). Ohmmeter.

AMMETER - VOLTMETER METHOD:



Measured resistance $(R_{m1}) = \frac{V}{I}$

$$R_{m1} = \frac{V_R + V_a}{I_R}$$

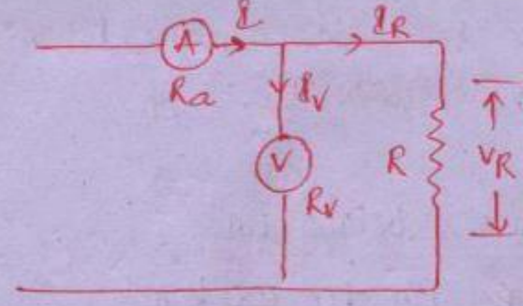
$$= \frac{I_R \cdot R + I_R \cdot R_a}{I_R} = R + R_a$$

True resistance $R = R_{m1} - R_a$.

$$\%\ \text{error} = \frac{A_m - A_t}{A_t}$$

$$= \frac{R_{m1} - R}{R}$$

$$= \frac{R_a}{R}$$



$$R_{m2} = \frac{V}{I} = \frac{V_R}{I_R + I_V} = \frac{V_R}{\frac{V_R}{R} + \frac{V_R}{R_V}}$$

$$\Rightarrow \frac{1}{R_{m2}} = \frac{1}{R} + \frac{1}{R_V}$$

$$\Rightarrow \frac{1}{R} = \frac{1}{R_{m2}} - \frac{1}{R_V}$$

$$R_{m2} = \frac{1}{\frac{1}{R} + \frac{1}{R_V}}$$

$$= \frac{1}{\frac{1}{R}\left[1 + \frac{R}{R_V}\right]}$$

$$= \frac{R}{1 + \frac{R}{R_V}}$$

$$\Rightarrow R_{m2}\left[1 + \frac{R}{R_V}\right] = R \quad \Rightarrow R_{m2} - R = -\frac{R_{m2} \cdot R}{R_V}$$

$$\frac{R_{m2} - R}{R} = -\frac{R_{m2}}{R_V}$$

$$\%\ \text{Error} = \frac{R_{m2} - R}{R}$$

$$= -\frac{R_{m2}}{R_V}$$

$$\simeq -\frac{R}{R_V}$$

V-A METHOD	A-V CONNECTION
* Measured resistance is always more than true resistance.	* Measured resistance is always less than the true resistance.
* This method of connection is suitable for measurement of high resistance in the specified band.	* This method of connection is suitable for measurement of low resistance
$\therefore \downarrow \%\ \text{Error} = \frac{R_a}{R \uparrow}$	$\therefore \downarrow \%\ \text{Error} \approx -\frac{R \downarrow}{R_V}$

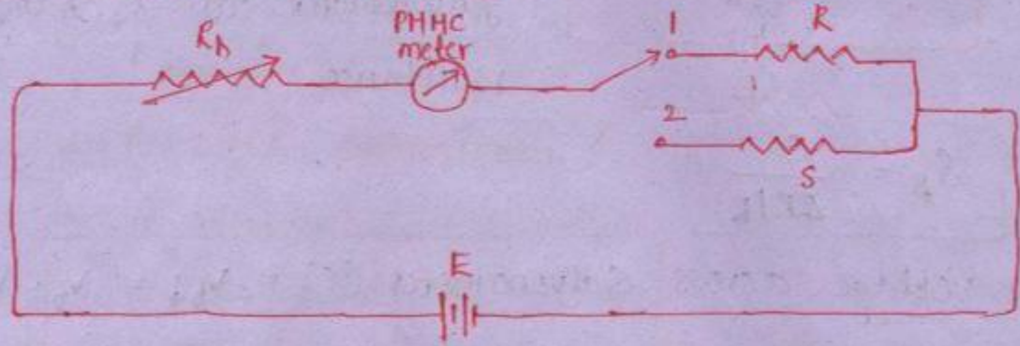
* Resistance for which both the methods gives equal error can be obtained as follows:

$$\frac{R_a}{R} = \frac{R}{R_V}$$

If R high adopt V-A conne.
If R is low → A-V connection

$$\Rightarrow \boxed{R = \sqrt{R_a R_V}}$$

SUBSTITUTION METHOD:



R - unknown

S - std known variable resistance

Keep the switch in (1) pos. and vary the R_h till a finite current passing through Ammeter. Change the pos. of switch to (2) and then vary the known resistance till same current passing through meter. In this R_h should not be disturb. At this condi. unknown resistance is equal to the known resistance.

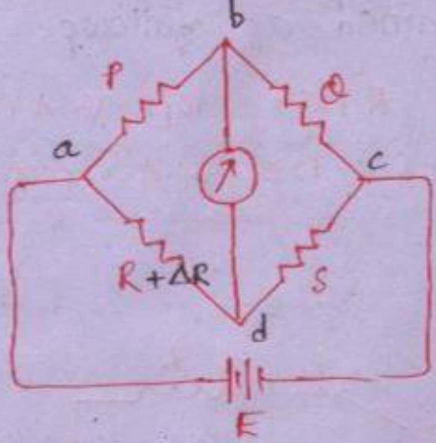
Let G = ~~un~~ Rheostat & meter resistance together

I_r = current through the meter whenever R is in circuit.

I_s = " " " S is in ckt.

$$\boxed{\frac{I_r}{I_s} = \frac{S+G}{R+G}}$$

WHEAT STONE BRIDGE :-



Under Balance condition,

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

* Bridge sensitivity is the ratio of deflection of galvonometer to % change in resistance.

$$\boxed{S_B = \frac{\theta}{\Delta R / R}}$$

Voltage across Galvonometer $(e) = V_{bd} = V_b - V_d$

$$= \left\{E - \frac{E}{P+Q}P\right\} - \left\{E - \frac{E}{R+\Delta R+S}(R+\Delta R)\right\}$$

$$= E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right]$$

$$= E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{R}{R+S}\right]$$

$$e = E\left[\frac{R^2 + RS + R\Delta R + S\Delta R - R^2 - R\Delta R - RS}{(R+S)^2 + \Delta R(R+S)}\right]$$

$$= E\left[\frac{S \cdot \Delta R}{(R+S)^2}\right]$$

Let S_V = Sensitivity of Galvonometer $= \frac{\theta}{e}$

$$\theta = S_V \cdot e = S_V \cdot \frac{E \cdot S \cdot \Delta R}{(R+S)^2}$$

$$S_B = \frac{\theta}{\Delta R / R} = \frac{S_V \cdot E \cdot S \cdot R}{(R+S)^2}$$

$$S_B = S_V \cdot E \cdot \frac{RS}{R^2 + 2RS + S^2}$$

$$\Rightarrow \boxed{S_B = \frac{S_V \cdot E}{\frac{R}{S} + 2 + \frac{S}{R}}}$$

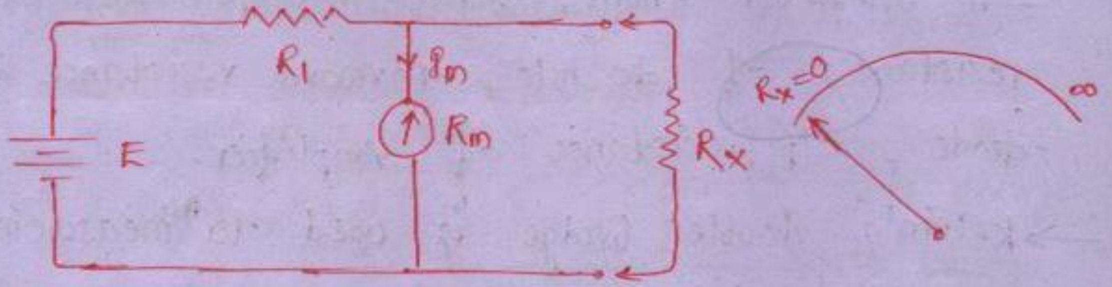
(or)

$$\boxed{S_B = \frac{S_V \cdot E}{\frac{P}{Q} + 2 + \frac{Q}{P}}}$$

Bridge sensitivity would be max. if $\frac{P}{Q} = \frac{R}{S} = 1$.

$$\Rightarrow \boxed{S_{B_{max}} = \frac{S_V \cdot E}{4}}$$

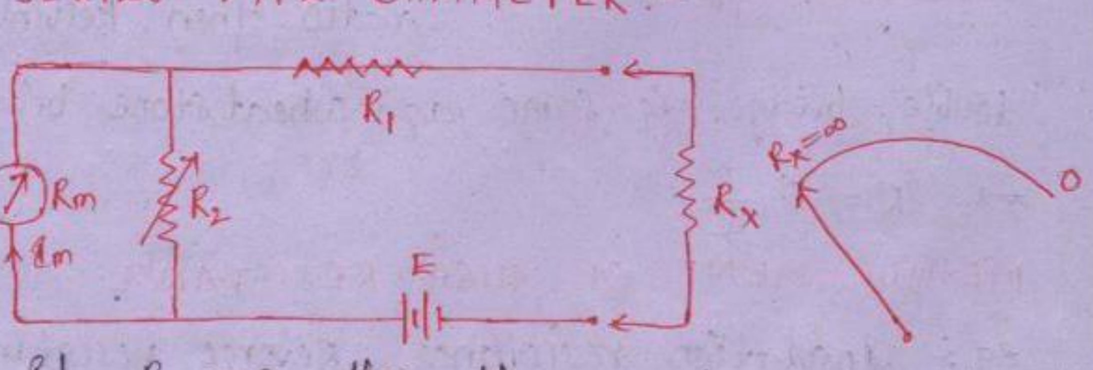
SHUNT TYPE OHMMETER:



If $R_x = 0$, the current passing through the meter is zero, hence the pointer occupies left most position on the scale.

If $R_x = \infty$, then max current will pass through meter, pointer occupies right most position on the scale.

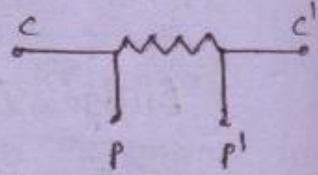
SERIES TYPE OHMMETER:-



If $R_x = \infty$, then the current passing through the meter is zero and it represents on left most on the scale.

If $R_x = 0$, current passing through the meter is max, then pointer deflects to right most.

MEASUREMENT OF LOW RESISTANCE:



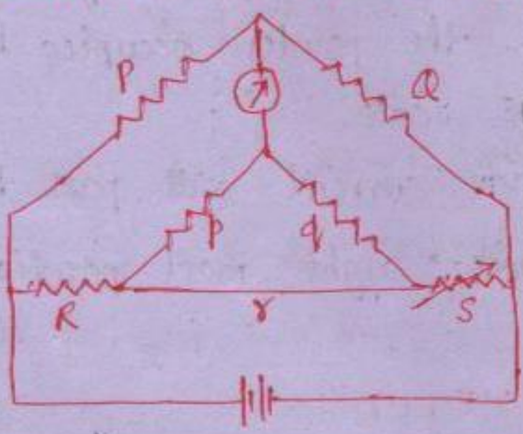
Low resistance is represented by 4 terminals; P & P' → voltmeter connection, C & C' → Ammeter connection.

These eliminates error due to leads & contacts.

eg:- Ammeter shunt, series, interpole and arm. resistances of dc m/c, forward resistance of diode, o/p resistance of Amplifier.

→ kelvin's double bridge is used to the measurement of low resistance.

KELVIN'S DOUBLE BRIDGE:



under balance condi.

$$\boxed{R = S \cdot \frac{P}{Q} + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]}$$

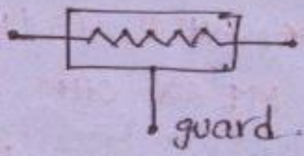
* If external arms ratio = inner arms ratio then kelvin's double bridge is same as wheatstone bridge.

$\rightarrow \quad R = S \cdot \frac{P}{Q}.$

MEASUREMENT OF HIGH RESISTANCE:

eg:- Insulation resistance, Reverse resistance of diode, i/p resistance of amplifier.

High resistance is represented by 3 terminals resistive element.



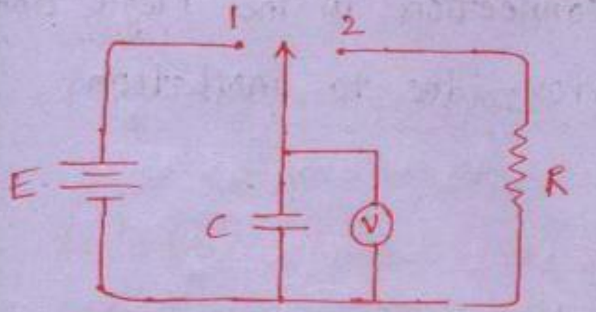
3rd terminal is guard terminal and it is useful for eliminating errors due to insulation

→ High resistance can be measured by

(1). Direct deflection method :-

This method is suitable for measurement of volume resistivity, surface resistivity of any insulating material available in sheet form.

(2). LOSS OF CHARGE METHOD:



capacitor gets charge towards supply voltage by connecting to pos. 1.

At pos. 2, capacitor start discharging through unknown resistance.

Let v be the voltage across capacitor after a finite time T.

$$e = E \cdot e^{-t/RC}$$

At $t = T$; $e = v$

$$v = E \cdot e^{-T/RC}$$

$$\Rightarrow \frac{v}{E} = e^{-T/RC}$$

$$\Rightarrow e^{T/RC} = \frac{E}{v}$$

$$\Rightarrow \frac{T}{RC} = \ln \frac{E}{v}$$

$$T = RC \cdot \ln \frac{E}{v}$$

($\tau = RC$)

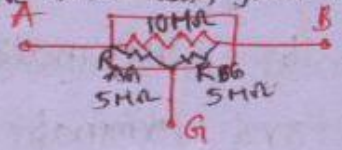
$$R = \frac{T}{C * 2.303 \log_{10} \frac{E}{v}}$$

$$\Rightarrow \boxed{R = \frac{T}{C \log_e \frac{E}{v}}}$$

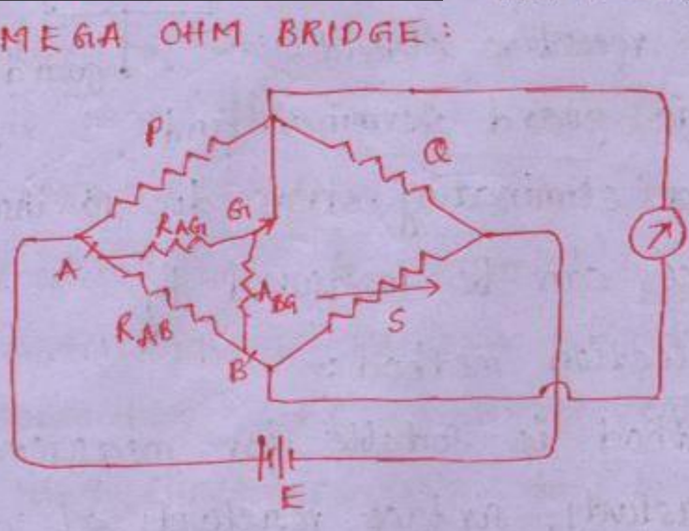
$$R = \frac{0.4343\ \cancel{0.4545}\ T}{C \log_{10} \frac{E}{v}}$$

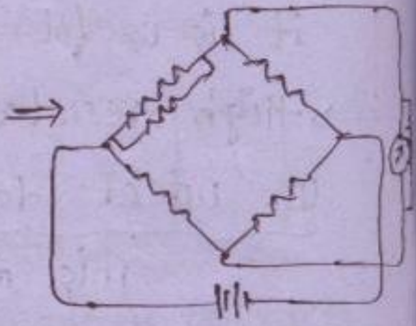
$$\boxed{\log_{10} x = 0.4343 \ln x}$$

If it is measured with wheatstone then it will give 5MΩ. but with guard → 10MΩ.



MEGA OHM BRIDGE:





(1). Wheatstone bridge with guard terminal is the Megaohm bridge.

(2). Guard terminal connection in the Mega ohm bridge eliminates error due to insulation resistance.

MEGGER:

(1). Operating voltage of the megger is more than the multimeter. It may be 500V, 1000V, 2000V, 5000V etc.

(2). Due to high operating voltage it results into finite value of current due to unknown resistance.

(3). Megger consists of self driven generator or pre charged capacitor for generating high voltage.

(4). Its principle based on Ratiometer Ohmmeter.

AC BRIDGES:

APPLICATIONS:

Measurement of indutance, capacitance, freq, Q-factor, D-factor, dielectric const. of insulating materials.

DETECTORS:

In wheatstone bridge → D'Arsenval Galvonometer.

(1). Vibration Galvonometer

(2). Head phones

(3). Tunable amplifier

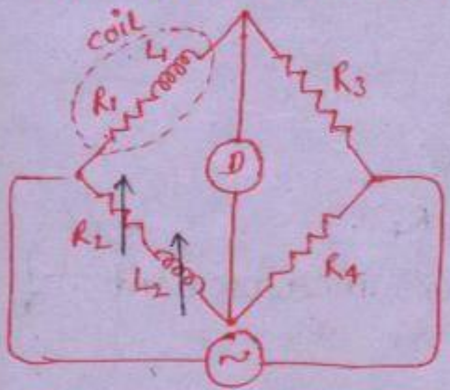
SOURCES:

power freq. AC source, Electronic oscillators

⇑ for low freq. ⇑ for more freq.

Inductance Bridges → $\boxed{\text{MAH/OM}}$

MAXWELL'S INDUCTANCE BRIDGE:



Under balanced condi:

$$\boxed{Z_1 Z_4 = Z_2 Z_3}$$

$$[R_1 + j\omega L_1] R_4 = [R_2 + j\omega L_2] R_3$$

Equate real parts,

$$R_1 R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4} = \frac{R_3}{R_4} \cdot R_2$$

Equating Ima. parts

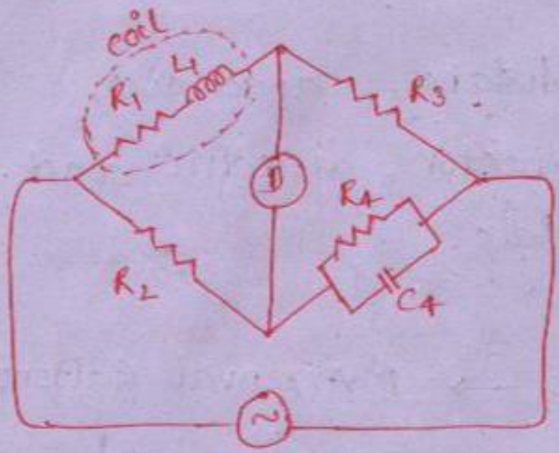
$$\omega L_1 R_4 = \omega_2 L_2 R_3$$

$$\Rightarrow L_1 = \frac{R_3}{R_4} \cdot L_2$$

Variable quanties : R_2, L_2

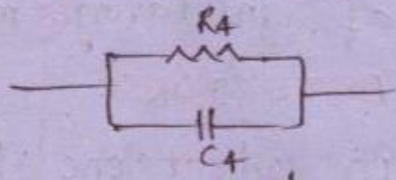
Balanced eq.s are ind. in nature hence balance can be achieved very easily

MAXWELL'S INDUCTANCE – CAPACITANCE BRIDGE:



under balance condition:

$$Z_1 Z_4 = Z_2 Z_3$$



$$Z = \frac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{R}{1 + j\omega RC}$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3$$

$$\Rightarrow (R_1 + j\omega L_1) R_4 = R_2 R_3 (1 + j\omega R_4 C_4)$$

Equate Real,

$$R_1 R_4 = R_2 R_3$$

$$\Rightarrow \boxed{R_1 = \frac{R_3}{R_4} \cdot R_2}$$

Equate imaginary,

$$\omega L_1 R_4 = \omega R_4 C_4 R_2 R_3$$

$$\Rightarrow \boxed{L_1 = R_2 R_3 \cdot C_4}$$

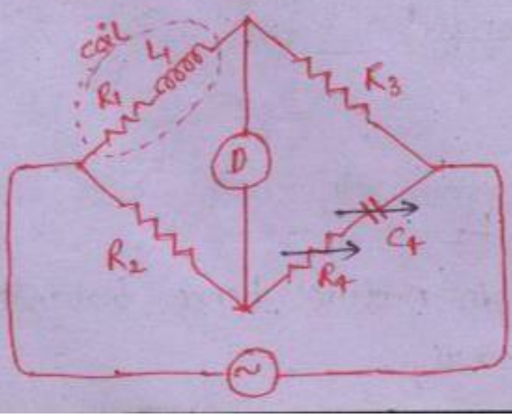
Q - factor: $\frac{\omega L_1}{R_1}$

$$= \omega R_2 R_3 C_4 \times \frac{R_4}{R_3 R_2}$$

$$\Rightarrow \boxed{Q - \text{factor} = \omega C_4 R_4.}$$

This bridge is suitable for measurement of low Q coils [Q < 10].

HAY'S BRIDGE:



under balance condition

$$Z_1 Z_4 = Z_2 Z_3.$$

$$(R_1 + j\omega L_1)\left(R_4 + \frac{1}{j\omega C_4}\right) = R_2 R_3$$

$$\Rightarrow (R_1 + j\omega L_1)(1 + j\omega R_4 C_4) = j\omega R_2 R_3 C_4$$

Equate real parts,

$$R_1 - \omega^2 L_1 R_4 C_4 = 0$$

$$\Rightarrow R_1 = \omega^2 L_1 C_4 R_4$$

Equate imaginary,

$$\omega L_1 + \omega R_1 R_4 C_4 = \omega R_2 R_3 C_4$$

$$\Rightarrow L_1 + R_4 C_4 (\omega^2 L_1 C_4 R_4) = R_2 R_3 C_4$$

$$\Rightarrow L_1 [1 + \omega^2 R_4^2 C_4^2] = R_2 R_3 C_4$$

$$\Rightarrow \boxed{L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}}$$

$$R_1 = \omega^2 C_4 R_4 \times \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}$$

$$\boxed{R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}}$$

Q - factor: $\frac{\omega L_1}{R_1}$

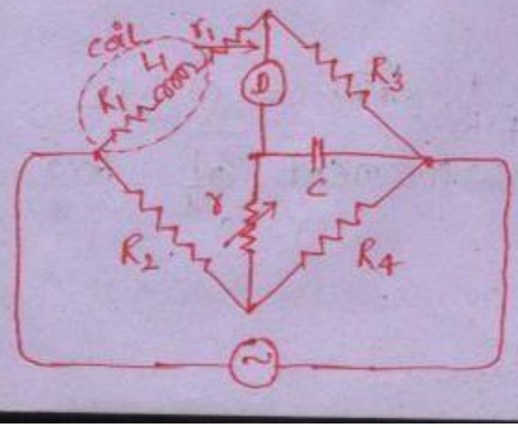
$$= \omega \cdot \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \times \frac{1 + \omega^2 R_4^2 C_4^2}{\omega^2 R_2 R_3 R_4 C_4^2}$$

$$\boxed{Q\text{-factor} = \frac{1}{\omega R_4 C_4}}.$$

It is suitable for the measurement of high Q- coils [Q > 10].

Balance eq.s are not ind. hence it is very diff to get the balance of the bridge.

ANDERSON'S BRIDGE:



under balanced condi.

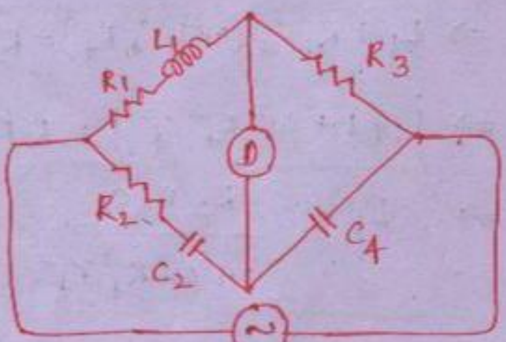
$$\boxed{R_1 = \frac{R_2 R_3}{R_4} - r_1}$$

$$\boxed{L_1 = C \frac{R_3}{R_4} \{ r(R_2 + R_4) + R_2 R_4 \}}$$

It is very difficult to derive the balance eq.s

It is very easy to achieve balance of bridge since it employs 2 variable resistances for getting balance.

OWEN'S BRIDGE:



This is only inductance bridge which consists of two capacitances.

under balance condi.

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = R_3\left[R_2 + \frac{1}{j\omega C_2}\right]$$

$$\Rightarrow \frac{R_1}{j\omega C_4} + \frac{L_1}{C_4} = R_2 R_3 + \frac{R_3}{j\omega C_2}$$

Equate real parts,

$$\frac{L_1}{C_4} = R_2 R_3$$

$$\Rightarrow \boxed{L_1 = R_2 R_3 C_4}$$

Equate imaginary,

$$\frac{R_1}{\omega C_4} = \frac{R_3}{\omega C_2}$$

$$\Rightarrow \boxed{R_1 = \frac{C_4}{C_2} \cdot R_3}$$

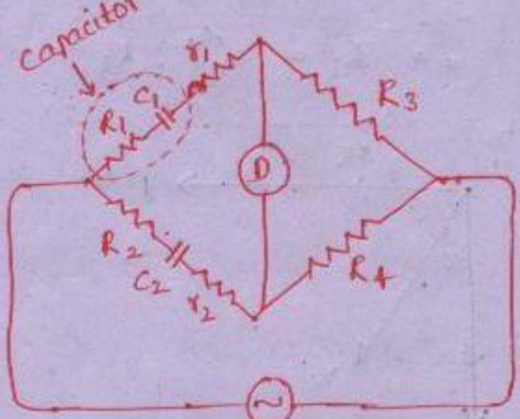
Q - factor $= \frac{\omega L_1}{R_1}$

$$= \omega \cdot \frac{R_2 R_3 C_4}{C_4 \cdot R_3} \cdot C_2 = \omega R_2 C_2$$

suitable for the measurement of low Q - coils.

CAPACITANCE BRIDGES:

DESAUTY'S BRIDGE:



under balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left[R_1 + r_1 + \frac{1}{j\omega C_1}\right] R_4 = \left[R_2 + r_2 + \frac{1}{j\omega C_2}\right] R_3$$

Equate real parts,

$$(R_1 + r_1) R_4 = (R_2 + r_2) R_3$$

$$\Rightarrow R_1 = \frac{R_3}{R_4}(R_2 + r_2) - r_1$$

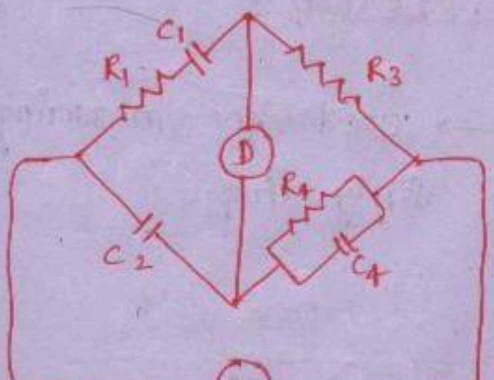
Equate imaginary,

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C_1 = \frac{R_4}{R_3} \cdot C_2$$

It is suitable for the measurement of practical capacitor.

SCHERING BRIDGE:



→ Having 3 capacitors.

under balance condi.

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left[R_1 + \frac{1}{j\omega C_1}\right]\left[\frac{R_4}{1 + j\omega R_4 C_4}\right] = \frac{R_3}{j\omega C_2}$$

$$\Rightarrow \left[\frac{1 + j\omega R_1 C_1}{\cancel{j\omega} C_1}\right]\left[\frac{R_4}{1 + j\omega R_4 C_4}\right] = \frac{R_3}{\cancel{j\omega} C_2}$$

$$\Rightarrow (1 + j\omega R_1 C_1) R_4 C_2 = R_3 C_1 [1 + j\omega R_4 C_4]$$

Equate real parts,

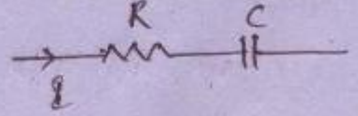
$$R_4 C_2 = R_3 C_1$$

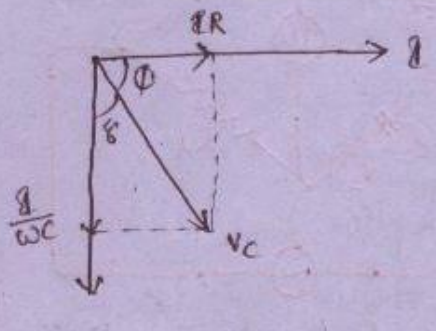
$$\Rightarrow \boxed{C_1 = \frac{R_4}{R_3} C_2}$$

Equate imaginary,

$$\omega R_1 R_4 C_1 C_2 = \omega R_3 R_4 C_1 C_4$$

$$\Rightarrow \boxed{R_1 = \frac{C_4}{C_2} \cdot R_3}$$





Loss angle $(\delta) = 90 - \phi$

$$D\text{-factor} = \tan\delta = \frac{IR}{I/\omega C}$$

$$\Rightarrow D\text{-factor} = \omega RC. \quad [\text{Dissipation factor}].$$

$$D\text{-factor} = \omega R_1 C_1 = \omega \cdot \frac{C_4}{C_2} \cdot R_3 \cdot \frac{R_4}{R_3} \cdot C_2 = \omega \cdot C_4 R_4$$

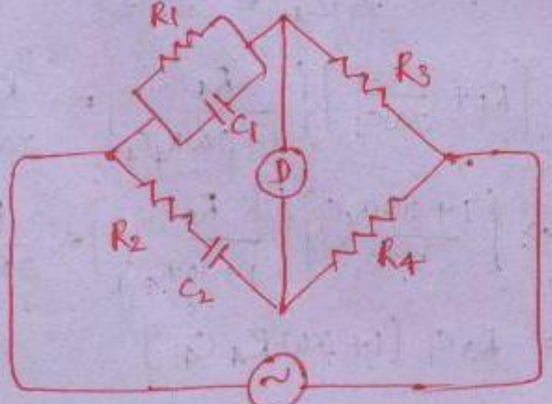
For the measurement of low capacitances, high voltage Schering bridge is preferable.

Wagner earth device is used to protect the operator under open circuit condition.

MEASUREMENT OF FREQUENCY :

WIEN'S BRIDGE :

→ used for measuring supply freq.



Under balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left[\frac{R_1}{1+j\omega R_1 C_1}\right] R_4 = \left[R_2 + \frac{1}{j\omega C_2}\right] \cdot R_3$$

$$\Rightarrow R_1 R_4 (j\omega C_2) = (1 + j\omega R_2 C_2) R_3 (1 + j\omega R_1 C_1)$$

Equate real parts,

$$0 = R_3 [1 - \omega^2 R_1 R_2 C_1 C_2]$$

$$\Rightarrow 1 - \omega^2 R_1 R_2 C_1 C_2 = 0.$$

$$\Rightarrow \omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}}\ ;$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$, then $f = \frac{1}{2\pi RC}$

(1). Harmonic distortion analyser

(2). Audio & high freq. oscillator.

MUTUAL INDUCTANCE BRIDGES:

(1). Heaviside Mutual Inductance bridge

(2). Cambell's Modification of Heaviside bridge

(3). Heaviside campbell's bridge.

(4). Carey foster bridge - Heydweiler bridge

3.1). $R_v = 500\,\Omega$

$R_a = 1\,\Omega$

$V = 20\,V$

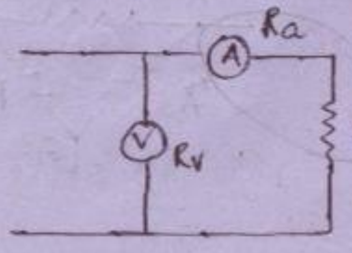
$I = 0.1\,A$

$R_{m1} = \frac{20}{0.1} = 200\,\Omega$

$R = R_{m1} - R_a$

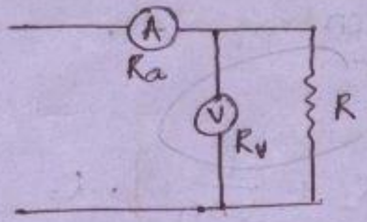
$= 199\,\Omega$.

(a).



Measured value $= R_{m1} = R + R_a$

(b).



$$R_{m2} = \frac{20}{0.1} = 200\,\Omega$$

par combi. $\boxed{\dfrac{1}{R_{m2}} = \dfrac{1}{R} + \dfrac{1}{R_V}}$

$$\Rightarrow \frac{1}{200} = \frac{1}{R} + \frac{1}{500}$$

$$\Rightarrow R = 333.33\,\Omega.$$

$$R = \sqrt{R_a R_V}$$

$$= \sqrt{1 \times 500}$$

$$= 22.36\,\Omega.$$

3.2)

$S = 100\,k\Omega$

$G = 2000\,\Omega$

$\theta_R = 46$ div

$\theta_S = 40$ div

$$\frac{\theta_R}{\theta_S} = \frac{S+G}{R+G}$$

$$\Rightarrow \frac{46}{40} = \frac{100+2}{R+2}$$

$$\Rightarrow R = 86.7\,k\Omega.$$

3.3)

$S = 500\,k\Omega$

$G = 10\,k\Omega$

$\theta_S = 41$; $\theta_R = 51$.

$$\frac{\theta_R}{\theta_S} = \frac{S+G}{R+G}$$

$$\Rightarrow \frac{41}{51} = \frac{R+10}{500+10}$$

$$\Rightarrow R = 400\,k\Omega.$$

3.4)

$P = 100\,\Omega$

$Q = 1000\,\Omega$

$p = 99.92\,\Omega$

$q = 1000.6\,\Omega$

$r = 0.1\,\Omega$

$S = 0.00377\,\Omega$

$$R = S \cdot \frac{P}{Q} + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$$

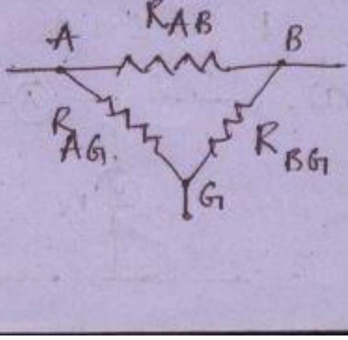
$$= \frac{100}{1000} \times 0.00377 + \frac{1000.6 \times 0.1}{99.92 + 1000.6 + 0.1} * \left[\frac{100}{1000} - \frac{99.92}{1000.6}\right]$$

$$= 389.7\,\mu\Omega$$

3.5)

$R_{AB} = 200\,M\Omega$

$R_{AG} = R_{BG} = 400\,M\Omega$



$$R_m = 200 \parallel [400 + 400]$$
$$= 160\ M\Omega$$
$$\%\ Error = \frac{A_m - A_t}{A_t} \times 100$$
$$= \frac{160 - 200}{200} \times 100$$
$$= -20\%.$$

3.6).

$C = 6 \times 10^{-4}\ \mu F$

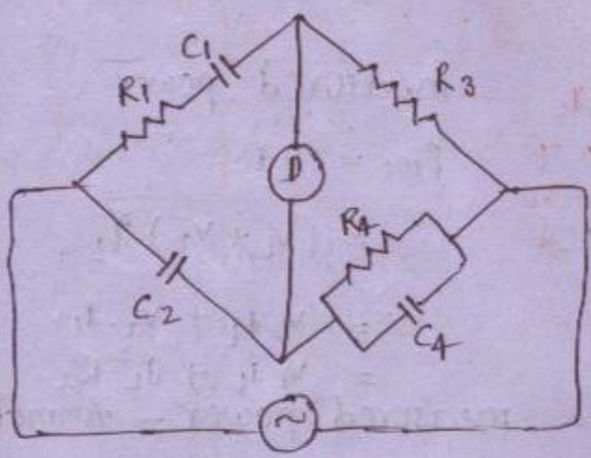
$E = 250V$

$v = 92V$

$t = 60$ sec.

$$R = \frac{t}{C \cdot \log_e\left(\frac{E}{v}\right)} = \frac{60}{6 \times 10^{-4} \times 10^{-6} \ln\left[\frac{250}{92}\right]}$$
$$= 1,00,000\ M\Omega.$$

3.8).



$C_2 = 106\ pF$

$R_4 = 1000/\pi\ \Omega$

$C_4 = 0.5\ \mu F$

$R_3 = 260\ \Omega$

$$R_1 = \frac{C_4}{C_2} \times R_3$$
$$= \frac{0.5 \times 10^{-6}}{106 \times 10^{-12}} \times 260$$
$$= 1.22\ M\Omega$$

$$C_1 = \frac{R_4}{R_3} \times C_2$$
$$= \frac{1000/\pi}{260} \times 106 \times 10^{-12}$$
$$= 1.29\ pF$$

$$pf = \cos\phi = \frac{R}{Z}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 129 \times 10^{-12}} = 24.6\,M\Omega$$

$$\tan\phi = \frac{X_c}{R}$$

$$= \frac{24.6}{1.22} \Rightarrow \phi = 87.16^\circ$$

$$pf = \cos(87.16)$$

$$= 0.05$$

$$C = \frac{\epsilon A}{d}$$

$$\Rightarrow 129 \times 10^{-12} = \frac{8.85 \times 10^{-12} \times \epsilon_r \times \frac{\pi (0.12)^2}{4}}{4.5 \times 10^{-3}}$$

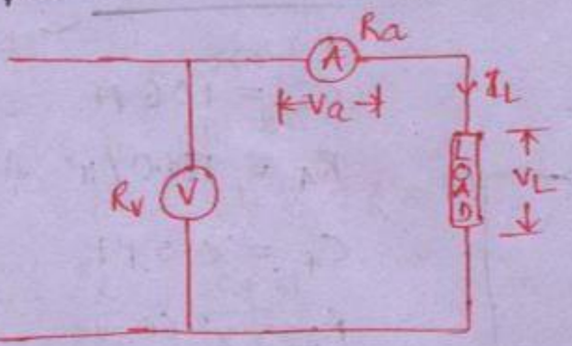
$$\Rightarrow \epsilon_r = 5.8$$

Author: problems by parker Smith

MEASUREMENT OF POWER:

MEASUREMENT OF POWER IN DC CIRCUIT:

* A-V method is useful to measure the power in DC ckts.



measured power

$$P_{m1} = VI$$

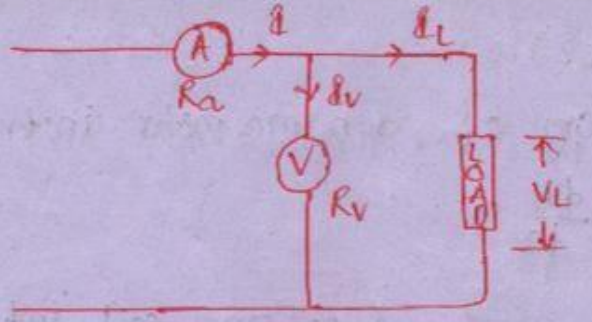
$$= (V_L + V_a) I_L$$

$$= V_L I_L + V_a \cdot I_L$$

$$= V_L I_L + I_L^2 R_a$$

$\Rightarrow$ * True power (P_t) = measured power − Ammeter power loss.

$$\Rightarrow P_t = P_{m1} - P_a.$$



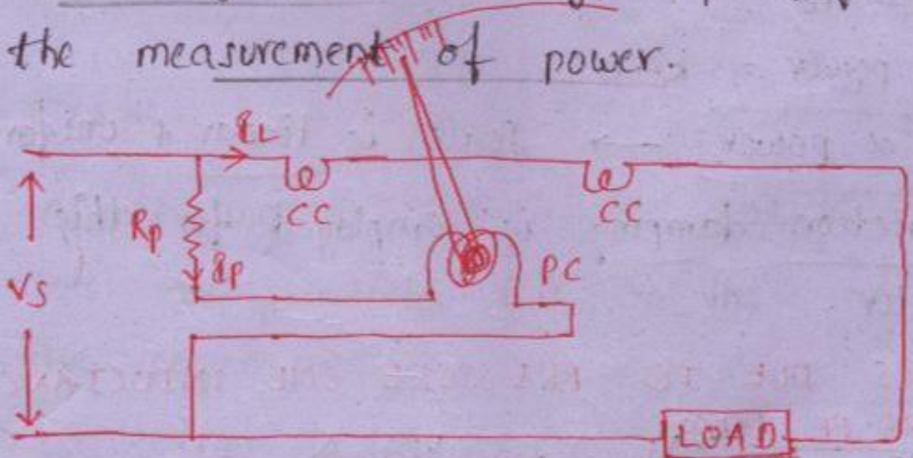
$$P_{m2} = VI$$
$$= V_L(I_L + I_v)$$
$$= V_L I_L + V_L \cdot I_v$$
$$= V_L I_L + V_L \cdot \frac{V_L}{R_v}$$
$$= V_L I_L + \frac{V_L^2}{R_v}$$

* True power = measured power − power loss in voltmeter.

* In both methods measured power is more than the true power. true power can be obtained by subtracting power loss of the meter connected near to load.

MEASUREMENT OF POWER IN 1-Ø AC:

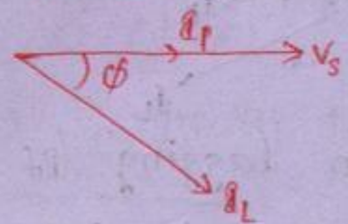
electro dynamo meter type is useful for the measurement of power.



CC - current coil

PC - pressure coil

$I_p = \frac{V_s}{R_p}$



wattmeter consists of 2 coils

-fixed coil is in series with load and carries load current hence it is known as CC.

Moving coil is connected across supply lines and carries current proportional to supply volt. hence it is known as PC.

from the theory of dynamo meter instr.

$$T_d = I_1 I_2 \cos\alpha \cdot \frac{dM}{d\theta}$$

$I_1 = I_L$

$I_2 = I_P = \frac{V_S}{R_P}$ [pressure coil inductance is neglected].

$\alpha = \phi$

$$\therefore \boxed{T_d = \frac{V_S}{R_P} \cdot I_L \cdot \cos\phi \cdot \frac{dM}{d\theta}}$$

$T_d \propto V_S I_L \cos\phi$

$T_d \propto$ power

Springs are used for control purpose, hence

$T_C \propto \theta \implies T_C = K_C\theta$.

At balanced condition,

$T_d = T_C$

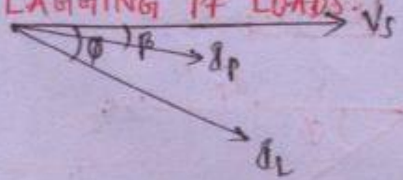
$\implies k_1 \cdot \text{power} = k_C \cdot \theta$

$\implies \boxed{\theta \propto \text{power}} \longrightarrow$ scale is linear & uniform.

* Air friction damping is employed for this wattmeter.

ERRORS DUE TO PRESSURE COIL INDUCTANCE:

LAGGING PF LOADS:



$I_P = \frac{V_S}{Z_P}$

$\cos\beta = \frac{R_P}{Z_P}$

On lagging pf loads, effective angles seen by currents (α) is < pf angle hence wattmeter shows more reading.

$\alpha = \phi - \beta$

wattmeter reading $(P_m) = I_L I_P \cos\alpha \cdot \frac{dM}{d\theta}$

$= I_L \cdot \frac{V_S}{Z_P} \cos(\phi - \beta) \cdot \frac{dM}{d\theta}$

$$= \frac{V_s I_L}{R_p} \cdot \cos\beta \cdot \cos(\phi-\beta) \cdot \frac{dM}{d\theta}.$$

True power $(P_t) = I_L \cdot I_p \cdot \cos\alpha \cdot \frac{dM}{d\theta}$:

$$= \frac{V_s \cdot I_L}{R_p} \cdot \cos\phi \cdot \frac{dM}{d\theta}.$$

$$\frac{\text{True power}}{\text{wattmeter reading}} = \frac{\frac{V_s \cdot I_L}{R_p} \cdot \cos\phi \cdot \frac{dM}{d\theta}}{\frac{V_s \cdot I_L}{R_p} \cdot \cos\beta \cdot \cos(\phi-\beta) \cdot \frac{dM}{d\theta}}.$$

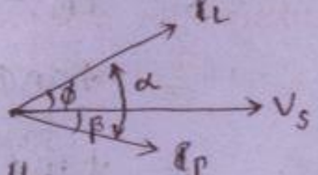
$$= \frac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)}$$

$$\Rightarrow \boxed{P_t = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)} \cdot P_m}$$

↑ correction factor.

On lagging pf loads, correction factor is always less than 1.

LEADING PF LOADS:



On leading pf loads, watt meter always shows less than the true power.

$\alpha = \phi + \beta$.

$$\boxed{\text{correction factor} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi+\beta)}}$$

On leading pf loads, CF is greater than 1.

$$\frac{P_t}{P_m} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)}$$

$$\simeq \frac{\cos\phi}{\cos(\phi-\beta)}$$

$$\simeq \frac{\cos\phi}{(\cos\phi \cdot \cos\beta + \sin\phi \sin\beta)}$$

$$= \frac{\cos\phi}{\cos\phi \cdot \cos\beta\,[1 + \tan\phi \tan\beta]}$$

$$\left[\frac{P_m}{P_t}\right]^{-1} = \frac{1}{1+\tan\phi\tan\beta} = \frac{P_t}{P_m}$$

$$\Rightarrow P_t\,[1 + \tan\phi \tan\beta] = P_m$$

$$\rightarrow P_m - P_t = \tan\phi \tan\beta \times P_t$$

$$\%\ \text{Error} = \frac{A_m - A_t}{A_t}$$

$$= \frac{P_m - P_t}{P_t}$$

$$= \tan\phi \tan\beta$$

$$\boxed{\%\ \text{Error} = \tan\phi \tan\beta.}$$

<u>Error</u> $= A_m - A_t$

$$= P_m - P_t$$

$$= \tan\phi \tan\beta \times \text{true power}$$

$$= \tan\phi \tan\beta \times V_s I_L \cdot \cos\phi$$

$$= \boxed{V_s \cdot I_L \cdot \sin\phi \tan\beta.}$$

FERRO DYNAMIC WATTMETER:

operating torque in electro dynamo meter is weak, due to presence of air cored coils.

So improve the strength of MF; iron cored coils may be employed. Then this meter is known as ferro dynamic wattmeter.

LOW pf WATTMETER :

$$T_d = \frac{V_s}{R_P} \cdot i_L \cdot \underset{(0.01)}{\cos\phi}$$

If normal wattmeter is employed for the measurement of power in low pf ckt then the amount of T_d is very less which may not be able to deflect the moving system.

⟹ The following modifications are suggested in low pf wattmeter.

(1). Reduce the resistance value connected in series with PC (R_p). Due to this T_d magnitude can be increased.

(2). By applying (employing) small control torque.

Hall effect multiplier is useful to generate an electrical signal (hall volt.) (V_H) ∝ to power consumption in the circuit. → {for Automatic Correction}

$V_H \propto K_H B i t$

where t = thickness of element

$B \propto$ voltage

$i \propto i_L$

$V_H \propto V_s \cdot i_L$

$\Rightarrow V_H \propto$ power.

wattmeter along with CT & PT is used for the measurement of large amount of power circuits.

* power = k × ratio of CT × ratio of PT × wattmeter reading

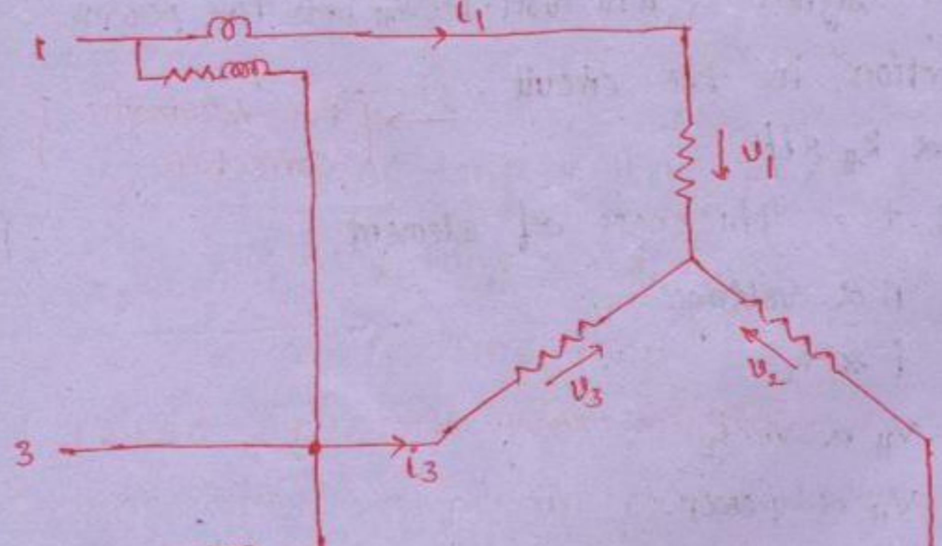
where $k = \dfrac{\cos\phi}{\cos\beta \cdot \cos(\phi \mp \beta)}$

MEASUREMENT OF POWER IN POLY PHASE CIRCUITS:

Blondel's Theorem is useful to decide the no. of wattmeters to be connected in the measurement of power of poly phase circuit.

THEOREM:

If a n/w is supplied through 'n' conductors, the total power is measured by summing the readings of 'n' wattmeters so arranged that current element of wattmeter is in each line and corr. voltage element is connected b/w that line and a common point, if the common point is located on one of the line then the power may be measured by 'n-1' wattmeters.



$P_1 \propto i_1 V_{13}$

$\propto i_1 (V_1 - V_3)$

$P_2 \propto i_2 V_{23}$

$\propto i_2 (V_2 - V_3)$

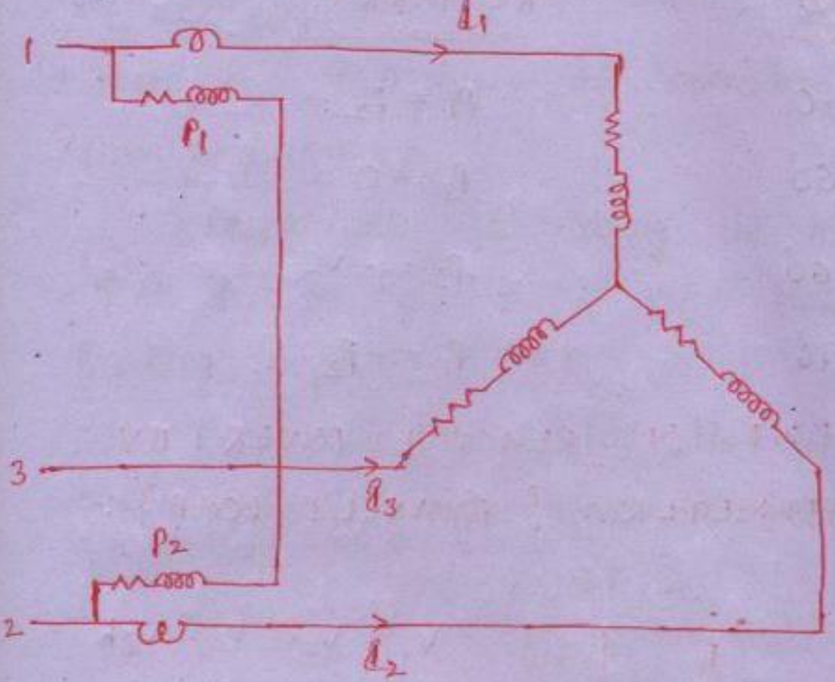
$P_1 + P_2 = i_1 (v_1 - v_3) + i_2 (v_2 - v_3)$

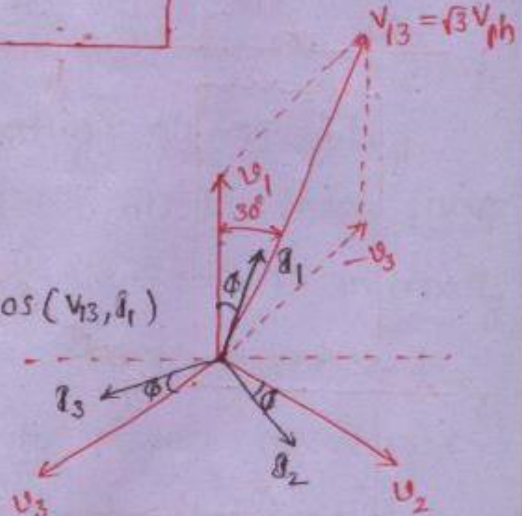
$= i_1 v_1 + i_2 v_2 - v_3 (i_1 + i_2)$

$$= v_1 i_1 + v_2 i_2 + v_3 i_3 \quad \{\because i_1 + i_2 + i_3 = 0\}$$

$$\Rightarrow P_1 + P_2 = P.$$

Two wattmeter method is suitable for measurement of power in both balanced & unbalanced 3-ϕ system.





$V_1 = V_2 = V_3 = V_{ph}$

$I_1 = I_2 = I_3 = I_{ph}$

wattmeter reading $P_1 = I_1 V_{13} \cos(V_{13}, I_1)$

$P_1 = I_{ph} \cdot \sqrt{3} \cdot V_{ph} \cdot \cos(30 - \phi)$

$P_1 = \sqrt{3}\, V_{ph} \cdot I_{ph} \cdot \cos(30 - \phi)$

wattmeter reading $P_2 = I_2 V_{23} \cdot \cos(V_{23}, I_2)$

$P_2 = \sqrt{3}\, V_{ph} \cdot I_{ph} \cdot \cos(30 + \phi)$

$$\boxed{\begin{aligned} P_1 + P_2 &= 3 \cdot V_{ph} \cdot I_{ph} \cdot \cos\phi \\ &= \text{total power} \\ \therefore\ P &= P_1 + P_2 \end{aligned}}$$

$P = \sqrt{3}\, V_L I_L \cos\phi$

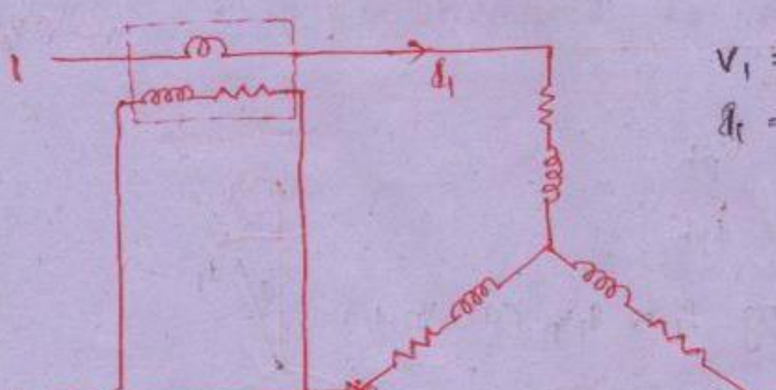
$$\boxed{\begin{aligned} P_1 - P_2 &= \sqrt{3} \cdot V_{ph} \cdot I_{ph} \cdot \sin\phi \\ \text{Reactive power } (Q) &= \sqrt{3} \cdot (P_1 - P_2) \end{aligned}}$$

$$\frac{Q}{P} = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$$

$$\Rightarrow \boxed{\tan\phi = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}}$$

pf	ϕ	REMARKS.
UPF	0	$P_1 = P_2$
0.5	60°	$P_2 = 0$
<0.5	$>60^\circ$	$P_2 = -ve$
0	90°	$P_1 = -P_2$

MEASUREMENT OF REACTIVE POWER BY SINGLE WATTMETER [BALANCED LOAD]:



$V_1 = V_2 = V_3 = V_{ph}$

$I_1 = I_2 = I_3 = I_{ph}$.

wattmeter reading $P = I_1 V_{23} \cos(V_{23}, I_1)$

$= I_{ph} . \sqrt{3} . V_{ph} . \cos(90 - \phi)$

$= \sqrt{3} . V_{ph} . I_{ph} . \sin\phi$

Reactive power $(Q) = \sqrt{3}$. wattmeter reading

MEASUREMENT OF ENERGY:

→ Integrating type instr-s are useful for the measurement of energy.
Motor meters are useful for the measurement of energy.

→ There are two types of operating torques:

Driving torque:-

Responsible for driving the moving system. [Al disc].

Braking torque:-

Responsible to control the disc movement and make it proportional to power consumption in case of energy meter.

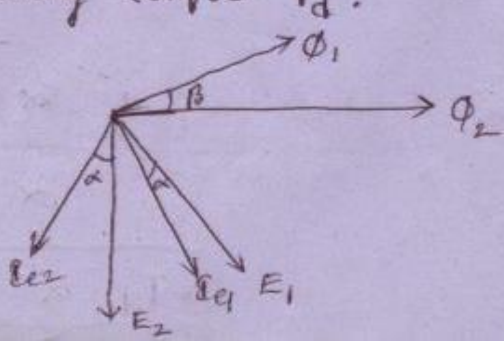
THEORY OF INDUCTION:

It consists of 2 alternating fluxes linking with common conducting media (disc), these fluxes produces 2 emf's (eddy emf's)., which internally circulates 2 eddy currents. first current I_1 & second flux ϕ_2 interact to produce one torque T_{d1}, second current I_2 and first flux ϕ_1 interact to produce another torque T_{d2}. These two torques acts in opp. dire to produce resultant torque known as driving torque T_d.

$e_1 \propto \frac{d\psi_1}{dt}$

$e_1 \propto \frac{d\phi_1}{dt}$

$\phi_1 = \phi_{m1} \sin\omega t$



$\phi_2 = \phi_{m_2} \sin(\omega t - \beta)$ $\qquad$ $e_2 \propto \frac{d\phi_2}{dt}$

$\Rightarrow e_1 \propto \phi_{m1}\, \omega \cdot \cancel{\sin}\cos\, \omega t$ $\qquad$ $\propto \phi_{m2}\omega \cdot \cos(\omega t - \beta)$

$E_1 \propto \frac{\phi_{m1}\omega}{\sqrt{2}}$ $\qquad$ $E_2 \propto \frac{\phi_{m2}\cdot\omega}{\sqrt{2}}$

$\propto \phi_1 \cdot 2\pi f$ $\qquad$ $\propto \phi_2 \cdot 2\pi f$

$E_1 \propto \phi_1 f.$ $\qquad$ $\propto \phi_2 \cdot f$

$I_{e_1} \propto \frac{\phi_1 f}{Z}$ $\qquad$ $I_{e_2} \propto \frac{\phi_2 f}{Z}$

Avg. deflection torque produced by interaction of I_{e1} & ϕ_2 is T_{d_1}.

$$T_{d1} \propto \phi_2\, I_{e1} \cos(\phi_2, I_{e1})$$

$$T_{d1} \propto \phi_2\, I_{e1} \cos(90 - \beta + \alpha) \Rightarrow T_{d_1} \propto \frac{\phi_1\phi_2 f}{Z}\cos(90-\beta+\alpha)$$

Avg. deflection torque T_{d_2} produced by interation I_{e2} & ϕ_1 is,

$$T_{d2} \propto \phi_1\, I_{e2} \cos(\phi_1, I_{e2})$$

$$\propto \phi_1\, I_{e2} \cdot \cos(90 + \beta + \alpha)$$

$$\propto \frac{\phi_1\, \phi_2 \cdot f}{Z} \cdot \cos(90 + \beta + \alpha)$$

Driving torque $T_d = T_{d_1} - T_{d_2}$

$$\propto \frac{\phi_1\phi_2 f}{Z}\{\cos(90 - \beta + \alpha) - \cos(90 + \beta + \alpha)\}$$

$$\propto \frac{\phi_1\phi_2 f}{Z}\{\sin(\beta - \alpha) + \sin(\beta + \alpha)\}$$

$$\propto \frac{\phi_1\phi_2 f}{Z} \cdot 2\sin\beta \cdot \cos\alpha$$

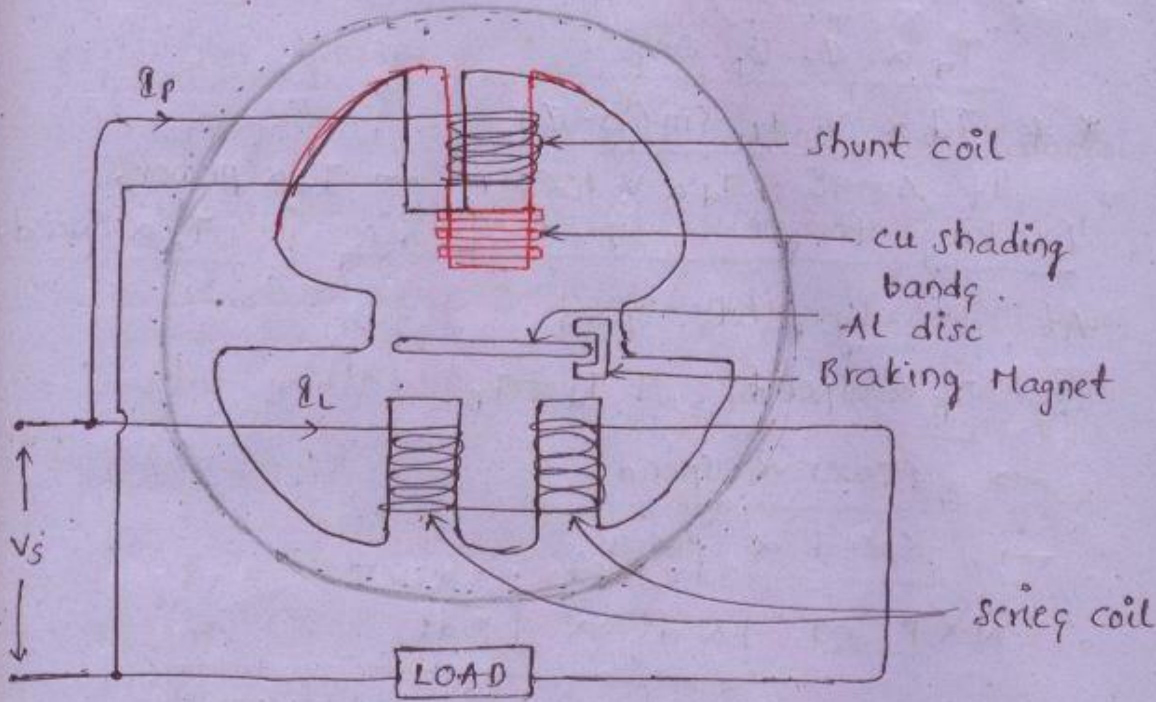
$$\propto \frac{\phi_1\phi_2 f}{Z} \cdot \sin\beta \cdot \cos\alpha$$

$$\boxed{T_d \propto \phi_1\, \phi_2 \sin\beta}$$

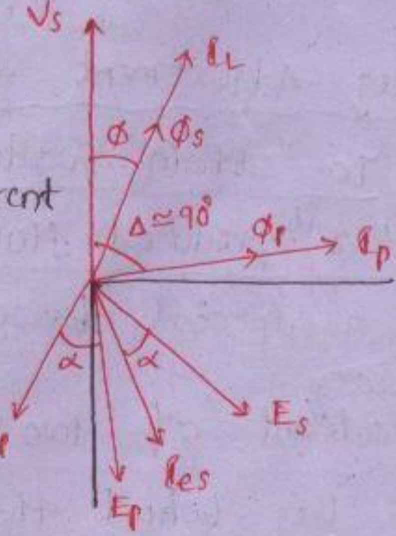
for Al disc path 'α' is fixed hence $\cos\alpha$ and impedance can be assumed to be const.

β - angle b/w two phases.

1-ϕ ENERGY METER:



Energy meter consists of,
coil in series with load and carries the load current is known as series coil.
coil which is connected across supply and draws current is known as shunt coil.



* current through shunt coil I_p lagging V by Δ nearly = 90° ($\Delta \simeq 90°$).

As per theory of Induction meter

$T_d \propto \phi_1 \phi_2 \sin\beta$.

$I_L \propto \phi_S \propto \phi_1$

$\phi_1 = \phi_S \propto I_L$.

$\phi_2 = \phi_P \propto I_P \propto V_S$.

$T_d \propto \phi_S \cdot \phi_P \cdot \sin\beta$

** $\boxed{T_d \propto V_S \cdot I_L \sin(\Delta - \phi)}$

If $\Delta = 90^\circ$; $T_d \propto V_S I_L \cos\phi$. $\rightarrow$ $T_d \propto$ power.

Braking torque $\propto$ Speed of disc ie $T_d \propto$ Speed.

At steady state speed,

$T_B \propto$ Speed ; $\because T_d = T_B$

$\Rightarrow$ power $\propto$ speed.

$\rightarrow$ $\boxed{\text{speed} \propto \text{power}}$

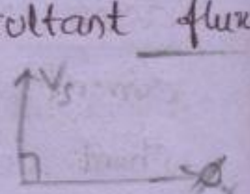
$\therefore N \propto P \Rightarrow \int N\,dt \propto \int P\,dt$

$\Rightarrow$ $\boxed{\text{no. of revolutions} \propto \text{energy}}$

Lag Adjustment devices :-

To obtain quadrature relation b/w supply volt & its produced flux, another flux is created by a special device known as lag adjustment device.

Resultant of two fluxes makes resultant flux 90° lag behind the supply voltage.



Eg: Cu shading bands

No. of revolutions made per kwh $\rightarrow$ meter const.

CREEPING:

Some times Energy meter shows more reading under low load conditions. Record some reading under NL condi. This is known as creeping.

→ Creeping is due to,

(1). Over compensation for friction

(2). Over voltage

→ Creeping can overcome by drilling to diametrically oppo. holes on Al disc.

→ Testing of EM is carried out by indirect loading metho phantom or fictitious loading.

4.1).

$R_C = 0.5\,\Omega$

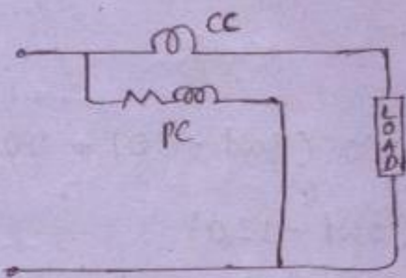
$R_P = 12500\,\Omega$

$pf = 1.$

$V_S = 250\,V$

$I_L = 4\,A$

(a).



power loss in CC $= I^2 R_c$

$= 4^2 (0.5)$

$= 8$ watts

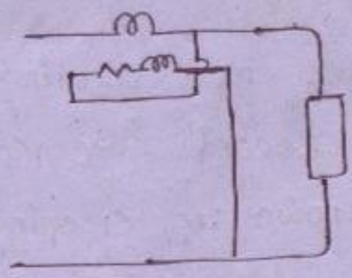
$\%\ \text{Error} = \dfrac{A_m - A_t}{A_t}$

$P_t = V_S I_L \cos\phi$

$= 250 \times 4 \times 1 = 1000\,\omega$

$\%\ \text{Error} = \dfrac{8}{1000} \times 100$

$= 0.8\ \%$



power loss in PC $= \frac{V^2}{R_p}$

$= \frac{250^2}{12500} = 5W.$

% Error $= \frac{5}{1000} \times 100$

$= 0.5\%$

4.2). $X_p = 0.01\, R_p$

(a). $\phi = \cos^{-1}(0.8)$

$= 36.86^\circ$

$\tan\beta = \frac{X_p}{R_p}$

$= 0.01$

% Error $= \tan\phi \tan\beta \cdot \times 100$

$= \tan 36.86^\circ \cdot \tan\beta \times 100$

$= 0.75$

(b). $\phi = \cos^{-1}(0.5) = 60^\circ.$

% Error $= \tan\phi \cdot \tan\beta \times 100$

$= 1.732\%$

(c). $\phi = \cos^{-1}(0.1) = 84.26^\circ$

% Error $= 9.95\%$

4.3).

$V = 100 \sin\omega t + 40 \cos(8\omega t - 30) + 50 \sin(5\omega t + 45)$

$i = 8 \sin\omega t + 6 \cos(5\omega t - 120)$

$$\boxed{P = \frac{1}{2\pi}\int_0^{2\pi} v \cdot i \cdot d(\omega t).}$$

* $\frac{1}{2\pi}\int_0^{2\pi} A \sin(\omega t + \alpha) \cdot B \cdot \sin(\omega t + \beta)\, d(\omega t)$

$= \frac{1}{2} AB \cdot \cos(\alpha - \beta).$

* $\frac{1}{2\pi}\int_0^{2\pi} A \sin(\omega t + \alpha) \cdot B \cos(\omega t + \beta)\, d(\omega t)$

$= \frac{1}{2} AB \cdot \sin(\alpha - \beta).$

* $\frac{1}{2\pi}\int_0^{2\pi} A\sin(m\omega t+\alpha).\ B\sin(n\omega t+\beta)\ d(\omega t) = 0.$

* $\frac{1}{2\pi}\int_0^{2\pi} A\cdot\sin(m\omega t+\alpha).\ B\cdot\cos(n\omega t+\beta)\ d(\omega t) = 0.$

$\therefore$ power $= \frac{1}{2}\times 100\times 8\ \cos(0) + 0 +$

$\frac{1}{2}\times 50\times 6\ \sin(165)$

$= 400 + 38.9 = 438.9\ \omega$

% fundamental $= \frac{400}{438.9}$

$= 91.13\ \%$

4.4). $P_1 = 5000\ \omega$

$P_2 = 1000\ \omega.$

$P = P_1 + P_2 = 6000\ \omega.$

$= \tan\phi = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$

$\Rightarrow \phi = 49.1^\circ$

$\therefore$ pf $= \cos 49.1 = 0.654$ lag

(b). $P_1 = 5000\ \omega$

$P_2 = -1000\ \omega$

$P = P_1 + P_2 = 4000\ \omega.$

$\tan\phi = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$

$\Rightarrow \phi = 69^\circ.$

pf $= \cos 69^\circ = 0.359$ lag.

4.5). pf $= 0.4$

$P = 30\ k\omega = P_1 + P_2$

$\phi = \cos^{-1}(0.4) = 66.4^\circ$

$$\tan\phi = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}$$

$$\Rightarrow \tan 66.4 = \frac{\sqrt{3}(P_1 - P_2)}{30}$$

$$\Rightarrow P_1 - P_2 = 39.68\,kW$$

$$P_1 + P_2 = 30\ kW.$$

$$\Rightarrow P_1 = 34.84\ kW$$

$$P_2 = -4.84\ kW.$$

4.6). $V_L = 400\,V$

$I_L = 30\,A.$

$P|_{ph} = V_{ph}.I_{ph}.\cos\phi = 5540\,W.$

$$\Rightarrow \frac{400}{\sqrt{3}} \times 30 \times \cos\phi = 5540$$

$$\Rightarrow \cos\phi = 0.8$$

$$\sin\phi = 0.6.$$

wattmeter reading $= \sqrt{3}.V_{ph}.I_{ph}.\sin\phi$

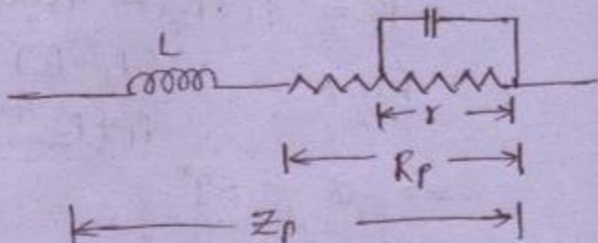
$$= \sqrt{3} \times \frac{400}{\sqrt{3}} \times 30 \times 0.6$$

$$= 7.2\ KVAR.$$

4.7).

COMPENSATION FOR PRESSURE COIL INDUCTANCE:

[WATTMETER]:



$$Z_p = R_p - r + jωL + \frac{r}{1 + jωrC}$$

$$= R_p - r + jωL + \frac{r(1 - jωrC)}{1 + ω^2 r^2 C^2}$$

for power freq.q $ω^2 r^2 C^2 << 1.$

$$\Rightarrow Z_p = R_p - r + jωL + r - jωr^2C.$$

$Z_p = R_p + j\omega[L - r^2 C]$

$\Rightarrow Z_p = R_p$ if $L = r^2 C$ $\left(\frac{L}{R} = RC\right)$

$\Rightarrow \boxed{C = \frac{L}{r^2}}$

4.7).

$C = 20\,pF$

$R_s = 10000\,\Omega$ ← series resistance in the pressure coil circuit.

$R_p = 400\,\Omega$.

$C = \frac{L}{r^2}$

$\Rightarrow 20 \times 10^{-12} = \frac{L}{(10000)^2}$

$\Rightarrow L = 2\,mH$.

4.8).

4.9).

$L_p = 8\,mH$

$R = 2000\,\Omega$

$\phi = 89°$

$f = 50\,Hz$.

$X_p = 2\pi f \cdot L_p$

$= 2\pi \times 50 \times 8 \times 10^{-3}$

$= 2.513\,\Omega$

% Error $= \tan\phi \cdot \tan\beta \times 100$

$\tan\beta = \frac{X_p}{R_p} = \frac{2.513}{2000} = 0.00125$

% Error $= \tan 89 \times 0.00125 \times 100$

$= 7.19\,\%$

4.10). $A_m = 250\,W = P_m$

$V = 200\,V$

$R_p = 2000\,\Omega$

True power = measured power − power loss in voltage coil.

P_x $\qquad$ P_m $\qquad$ $\frac{V^2}{R_p}$

$$= 250 - \frac{200^2}{2000}$$

$$= 230\ \omega.$$

5.1) Rating = 220V, 5A.

3275 rev/kwh ← meter constant.

power = 220×5 = 1100 ω.

Let FL is taken for 1 hr. then energy

consumption = $\frac{1100}{1000} \times 1 = 1.1$ kwh

No. of revolutions = 3275×1.1 = 3602.5 rev.

Speed = $\frac{3602.5}{3600} \simeq 1$ rps.

$I = 2.5$ A

$t = 59.5$ sec

30 revolutions

Energy = $\frac{220 \times 2.5}{1000} \times \frac{59.5}{3600}$

$= 0.00909$ kwh

No. of revolutions to be made = 0.00909 × 3275

$= 29.77$

% Error = $\frac{30 - 29.77}{29.77} \times 100 = 0.77$ % fast

5.2) $V = 230$V

$I = 50$A

61 revolutions

$t = 37$ sec.

meter constant = 520 rev/kwh

$$\text{Energy} = \frac{230 \times 50}{1000} \times \frac{37}{3600}$$

$$= 0.11819 \; \omega$$

No. of revolutions to be made $= 0.11819 \times 520$

$= 61.4.$

$$\%\ \text{Error} = \frac{61 - 61.4}{61.4} \times 100 = 0.65\%\ \text{slow}.$$

5.3).

$V = 250V$

$I = 15A$

$t = 5$ hrs

$pf = 1.$

$$\text{Reading} = 8253.13 - 8234.21$$

$$= 18.92 \; kwh$$

$$\text{Energy} = \frac{250 \times 15}{1000} \times 5 = 18.75 \; kwh$$

$$\%\ \text{Error} = \frac{18.92 - 18.75}{18.75} \times 100$$

$$= 0.9\%\ \text{high}$$

Revolutions $= 290$

$t = 5$ min

$I = 20A$

$V = 250V$

$pf = 0.87$

$$\text{Energy} = \frac{250 \times 20 \times 0.87}{1000} \times \frac{5}{60}$$

$$= 0.3625 \; kwh$$

$$\text{meter constant} = \frac{290}{0.3625} \Rightarrow 800$$

5.4). $\Delta = 87^\circ$

fL, upf $\longrightarrow N = 40$

$\frac{1}{4}$ fL, 0.5 pf lagging

$T_d \propto V_s \cdot I_L \sin(\Delta - \phi)$

$T_B \propto N$

$N \propto V_s I_L \sin(\Delta - \phi)$

$I_1 = I$ $\quad\quad$ $I_2 = I/4$

$\phi = 0$ $\quad\quad$ $\phi = 60^\circ$

$N_1 = 40$ $\quad\quad$ $N_2 = ?$

$$\frac{N_2}{N_1} = \frac{I_{L2} \cdot \sin(\Delta - \phi_2)}{I_{L1} \sin(\Delta - \phi_1)}$$

$$\Rightarrow \frac{N_2}{40} = \frac{I/4 \cdot \sin(87 - 60)}{I \cdot \sin(87 - 0)}$$

$$\Rightarrow N_2 = 40 \times \frac{1}{4} \cdot \frac{\sin 27}{\sin 87}$$

$$= 4.54.$$

5.5). $\Delta = 90^\circ$

$I_1 = I$ $\quad\quad$ $I_2 = I/4$

$\phi = 0$ $\quad\quad$ $\phi = 60$

$N_1 = N$ $\quad\quad$ $N_2 = ?$

$$\frac{N_2}{N_1} = \frac{I/4}{I} \cdot \frac{\sin(90 - 60)}{\sin(90 - 0)}$$

$$\Rightarrow N_2 = 0.125 N_1$$

$\Delta = 87^\circ$

$$\frac{N_2}{N_1} = \frac{I/4}{I} \cdot \frac{\sin(87 - 60)}{\sin(87 - 0)}$$

$$\Rightarrow N_2 = 0.113 N_1$$

$$\% \text{ Error} = \frac{0.113 N_1 - 0.125 N_1}{0.125 N_1} \times 100$$

$$= 9\% \text{ slow}$$

MEASUREMENT OF FREQUENCY:

Freq. Meters:

(1). Mech. resonance

(2). Electrical resonance

(3). weston type

(4). Ratio meter type

(5). Saturable core type

MECHANICAL RESONANCE TYPE:

Operating principle is based on mech. resonance. It mainly consists of thin steel trips (reeds) The electromagnet has laminated iron core and its coil is connected across the supply whose freq. is to be measured.

All reeds have slight diff in dimensions and weights due to which these exhibits diff in natural freq

* In unpolarized freq. meter the reed whose freq. is 2 times the supply freq. will make more vibrations.

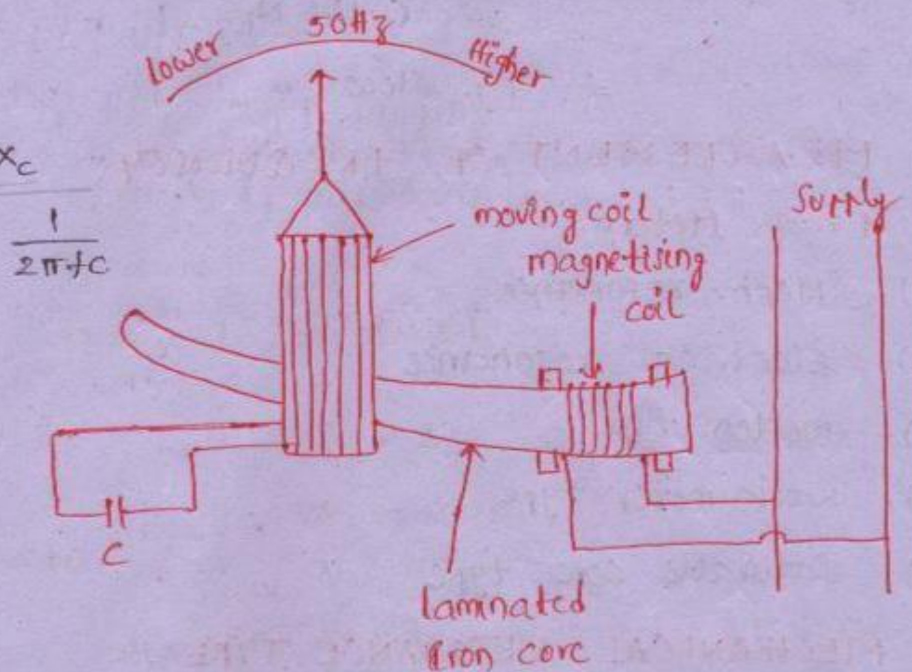
* In polarized freq. meter the reed whose natural freq is same as supply freq make more vibrations.

Range: 6 Hz [47 Hz to 53 Hz].

ELECTRICAL RESONANCE FREQ. METER:

$$X_L = X_C$$

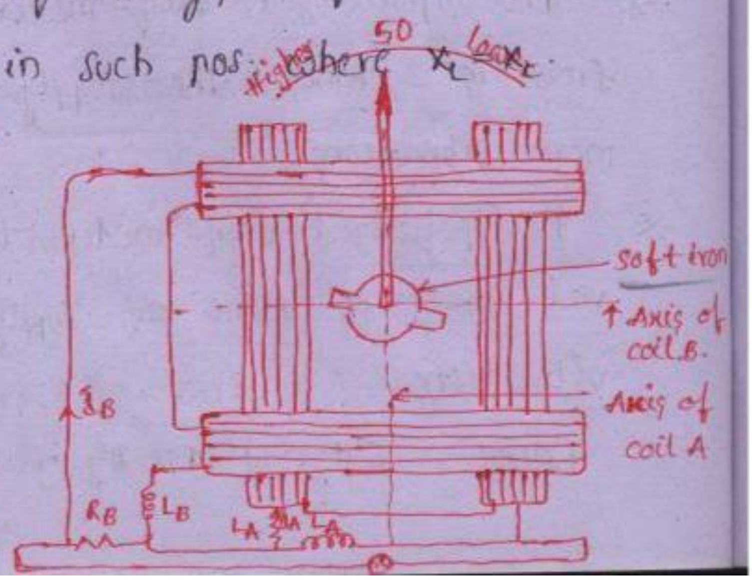
$$2\pi f L = \frac{1}{2\pi f C}$$



Operating principle based on electrical resonance. The magnetising coil mounted on laminated iron core whose cross section varies gradually over the length, being max near the end where magnetizing coil mounted and min at the other end. The inductance offered by moving coil is variable which depends on pos. of moving coil on iron core. for 50 Hz freq, coil occupies central location. As and when freq changes torque develop which brings moving coil in such pos. where X_L ...

* SAT. 22/11/08 *

WESTON TYPE:



The meter consists of 2 coils mounted $\perp^{er}$ to each other. Each coil is divided into 2 parts. coil A is connected in series with Resistance R_A and whole setup placed across L_A.

coil B is connected in series with inductance and whole ~~current~~ arrangement placed across L_B.

for a normal freq, pointer takes vertical -pos. As n when freq varies there will be diff in the magnitudes of i's through coils based mag. of i pointer takes up new pos. This indicates present value of freq.

RATIOMETER TYPE:

It consists of ratio meter which gives a linear relation b/w current ratio and deflection.
It is suitable for wide range of voltages.
It may be used for a freq range upto 5000 Hz.

SATURABLE CORE TYPE:

It is perticularly suitable for tachometer system.
suitable for wide range of freq.s.

MEASUREMENT OF POWER FACTOR:

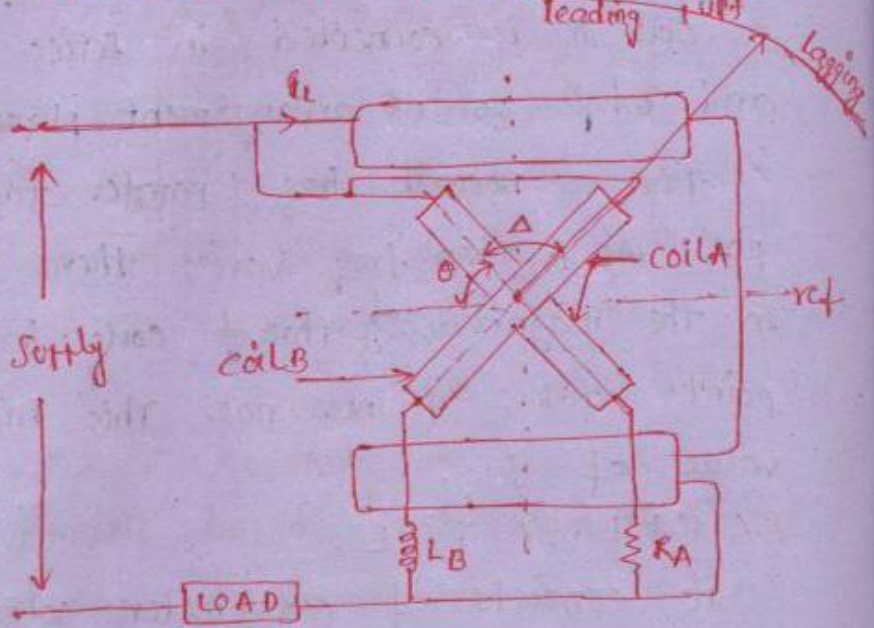
Pf meter indicates directly by a single reading, the pf of the ckt to be measured.
The moving system of pf meter is perfectly balanced at equilibrium by 2 opposing forces
∴ There no need for control torque.

2 Types:

(1). Electro dynamo meter type.

(2). Moving iron type.

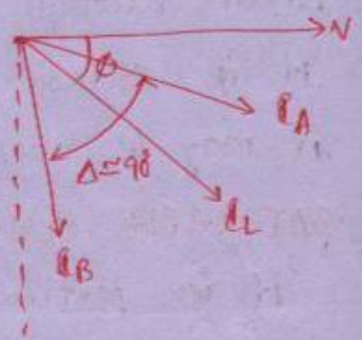
1-Ø ELECTRO DYNAMOMETER TYPE P.F.M:



At Supply freq,

$R_A = \omega L_B$.

Hence $I_A = I_B$, but both currents displaced by very near to 90° ($\Delta \simeq 90°$).



By the interaction of I_A, I_L torque will be generate on coil A and I_L & $I_B \longrightarrow$ torque will be on coil B.

Deflecting torque on coil A = $T_{dA} = I_1 I_2 \cos\alpha \frac{dM}{d\theta}$

$$\rightarrow T_{dA} = I_L \cdot I_A \cdot \cos(\phi) \frac{dM}{d\theta} \rightarrow M_{max} \sin(\theta)$$

$$\Rightarrow T_{dA} \propto I_L I_A \cos\phi \cdot M_{max} \cdot \sin\theta \cdot$$

Deflecting torque on coil B = T_{dB}

$$T_{dB} \propto I_L I_B \cos(90-\phi) \cdot M_{max} \cdot \sin(90+\theta)$$

At final deflection state,

$$T_{dA} = T_{dB}$$

$$\Rightarrow \cos\phi \cdot \sin\theta = \sin\phi \cdot \cos\theta$$

This eq. is satisfied when $\theta = \phi$.

At final deflection state, deflecting angle = power factor angle of the ckt.

For measurement of p.f. in 3-ϕ balanced load, same construction is suitable, but the angle b/w their planes is 120°. Fixed coil has to be connected in one line and 2 moving coils are connected across this line and other 2 lines individually.

For 3-ϕ unbalanced load p.f. measurement a 2 element p.f. meter is to be used.

MOVING IRON P.F.M:

(1). There are 2 types of moving iron p.f. meters.

(a). Rotating field P.F.M.

(b). Alternating field P.F.M. [Nalder Lip mann type]

In this meter also at steady state condi. $\theta = \phi$. The deflection of iron beam is direct measure of ph. angle b/w each line element and corr. ph. voltage.

Adv:

(1). The working forces are very large as compared with those electrodynamo meter type.

(2). All the coils in MI are fixed.

DIS . Adv :

(1). errors are introduced due to losses in iron parts.

(2). Calibration of these instr.s is effected variations in supply freq, volt & wave form.

POTENTIOMETERS

It is an instr. designed to measure an unknown volt. by comparing it with a known volt. This method is very accurate if volt. of ref source is accurately known.

No current flows under balance condi, hence no power consumption during measurement.

Determination of volt. by a potentiometer is ind. of source resistance.

Applications:

Calibration of Ammeter, volt. meter, measurement of current & voltage etc.

BASIC POTENTIOMETER CIRCUIT:

200 cm

$1\,cm = 1\Omega$

$101.86\,cm = 101.86\,\Omega$

$I = \frac{1.0186}{101.86}$

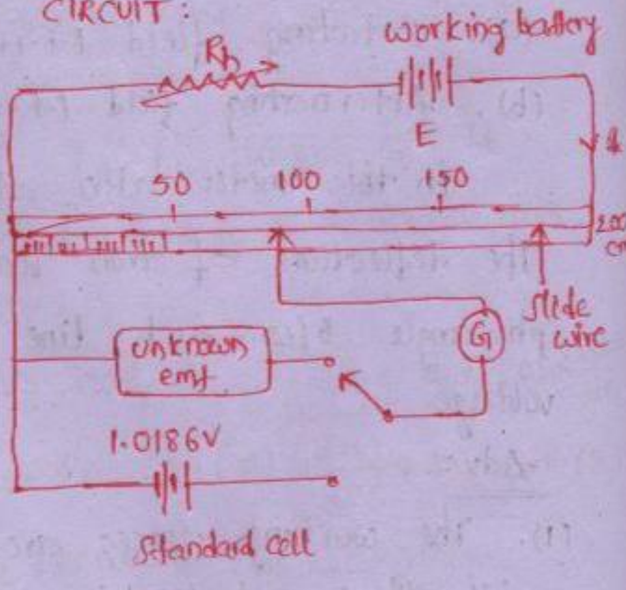
$= 10mA = 0.01A$

Eg 150 cm

$R = 150\Omega$

$V = 150 \times 0.01$

$= 1.5V.$

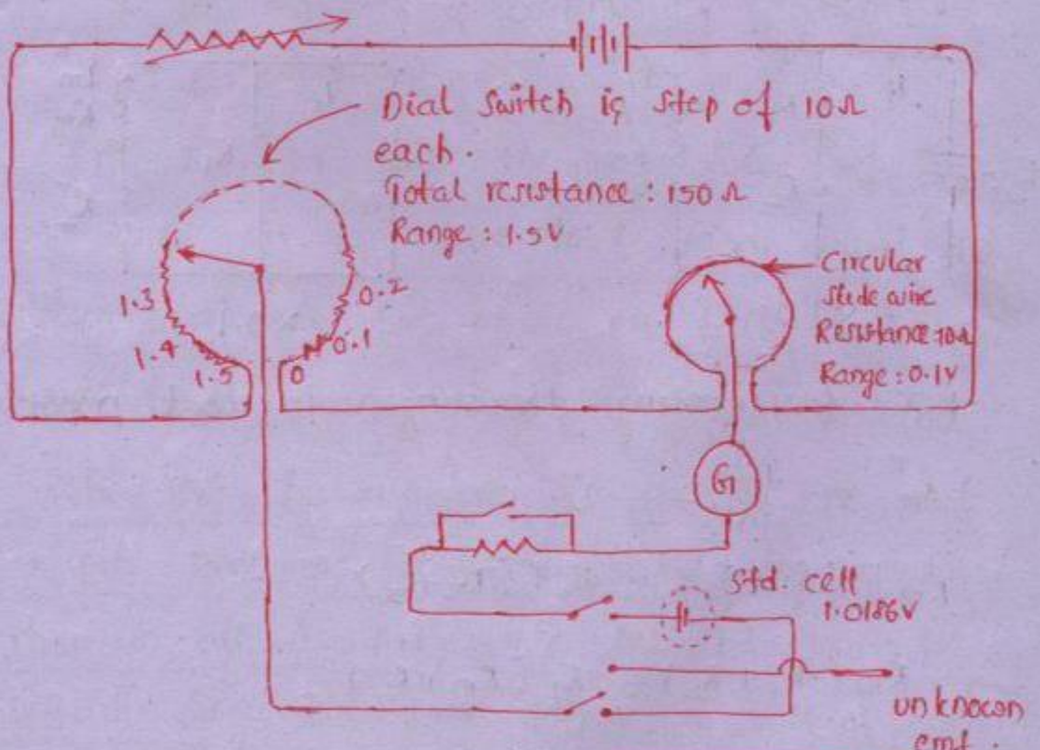


potentiometer is required to be standardised before making measurement by a std. cell.

keep the pos. of slider at 101.86 cm and switch at standardaised pos. Adjust until 'G' shows zero reading. while measuring unknown volt. slider is to be varied for its pos. till G shows zero reading.

unknown volt. can be found from the balance length and calibrated current.

LABORATORY TYPE [CROMPTON'S P.M.]:

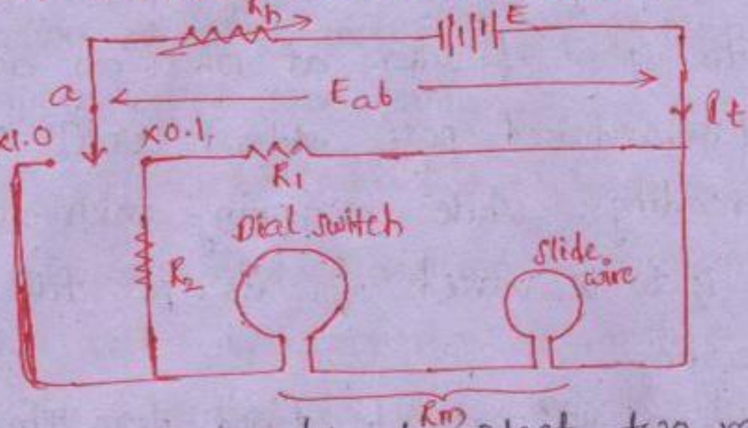


straight slide wire in basic potentiometer is replaced by deal switch & circular slide wire.

circular slide wire provided with 200 cm divisions hence min. volt. (resolution) measured is $\frac{0.1}{200} = 0.5mV$

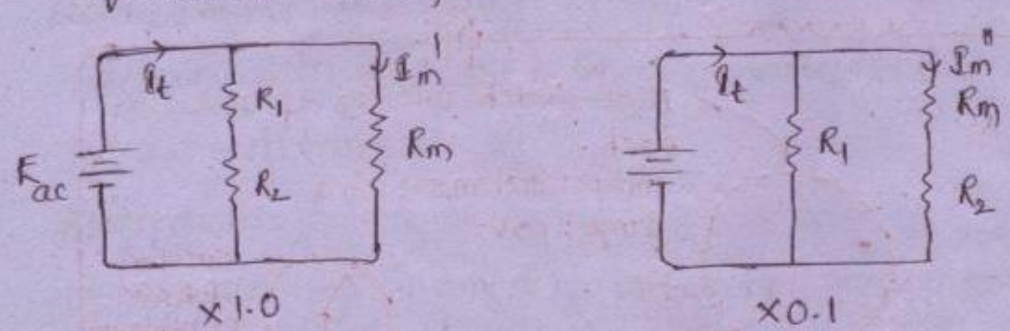
It is very easy to extend range of potentiometer by adding required no. of dials without disturbing calibrations.

MULTI RANGE POTENTIOMETER:



This P.M is useful to select two ranges without disturbing the calculations.

Equivalent circuits :-



for facilitating the ranges in that proportion,

$$I_m'' = \frac{I_m'}{10} \quad \text{——} (1)$$

$$R_m \parallel (R_1 + R_2) = R_1 \parallel (R_m + R_2)$$

$$\Rightarrow R_m (R_1 + R_2) = R_1 (R_m + R_2)$$

$$\Rightarrow R_1 = R_m$$

$$I_m'' = \frac{I_m'}{10}$$

$$\Rightarrow I_t \times \frac{R_1}{R_1 + R_2 + R_m} = \frac{1}{10} \times I_t \times \frac{R_1 + R_2}{R_1 + R_2 + R_m}$$

$$\Rightarrow R_2 = 9 R_1$$

$$\rightarrow R_2 = 9 R_m.$$

All the resistors in potentiometer are made up of MANGANIN (expect slide wire).

Slide wire is made up of from platinum silver alloy.

Sliding contact $\longrightarrow$ cu-gold-silver alloy.

There are 2 types of potentiometers:

(1). Vernier p.M.

(2). Brook's deflectional PM.

Brook's d.P.M is used for applications where volt. to be measured is continuously changed.

AC POTENTIOMETER:

In DC PM only the magnitudes of unknown emf and PM drop have to be made equal to obtain balance, but in AC PM both mag & ph. have to be made same to obtain balance.

The freq & wave form in the PM ckt must exactly same as that of volt. being measured. Thus in all ac PM's the PM ckt most be supplied from the same source as volt or current being measured.

A vibrational galvonometer is used as detector. standardaization of ac PM's can be done with the help of std DC source & transfer instr.

2 types of AC PM's:

(1). polar type PM :-

In these instr. the mag of unknown voltage is read from one scale and ph. angle w.r.t some ref phasor from a 2nd scale.

(2). Co-ordinate type: [Gall - Tinsley PM].

These instr.s are provided with 2 scales to read res.ly in phase comp & quadrature component of unknown voltage.

If higher voltages are to be measured a precision volt. divider called "volt-ratio box."

6.5). $V = 1.0185\ V$

$l = 50\ cm.$

(a). Emf of cell $= \frac{1.0185}{50} \times 72$

$= 1.467\ V.$

(b). $A_m = 1.33\ V$

$l = 64.5$

$A_t = \frac{1.0185}{50} \times 64.5 = 1.314\ V$

% Error $= \frac{1.33 - 1.314}{1.314} \times 100$

$= 1.22\%$

(c). $A_m = 0.43\ A$

voltage $= \frac{1.0185}{50} \times 43.2$

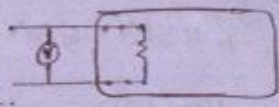
$= 0.8802\ V$

$A_t = \frac{0.8802}{2} = 0.4401\ A$

% Error $= \frac{0.43 - 0.4401}{0.4401} \times 100$

$= -2.29\%$

6.6). P.M $\longrightarrow$ 1V

$R_V = 10000 \times 5 = 50000\ \Omega$. 

with The connection of voltmeter, volt has become half hence the resultant resistance is also half. It is possible only if resistance of meter = internal ckt resistance.

$\therefore$ Resistance of ckt = 50,000 Ω.

$\Longrightarrow$ * INSTRUMENTATION TRANSFORMERS *

These T/F's are used in conjuction with meters for the measurement of high current & high voltage.

2 Types:

(1). Current T/f:

It will scaled down the current.

(2). potential T/f:

It will scaled down the voltage.

Transformation ratio: (R)

Ratio of primary phasor to secondary phasor.

$$R = \frac{I_P}{I_S}\Big|_{CT} \qquad R = \frac{V_P}{V_S}\Big|_{PT}$$

Nominal ratio: (k_n)

Ratio of rated primary phasor to rated secondary phasor.

$$k_n = \frac{\text{rated } I_P}{\text{rated } I_S}\Big|_{CT} \qquad k_n = \frac{\text{rated } V_P}{\text{rated } V_S}\Big|_{PT}$$

Turns ratio: (n)

$$n = \frac{N_S}{N_P}\Big|_{CT} \text{ s/up T/f} \qquad n = \frac{N_P}{N_S}\Big|_{PT} \text{ s/down T/f} \quad (n>1)$$

Ratio correction factor (RCF) = $\frac{R}{k_n}$.

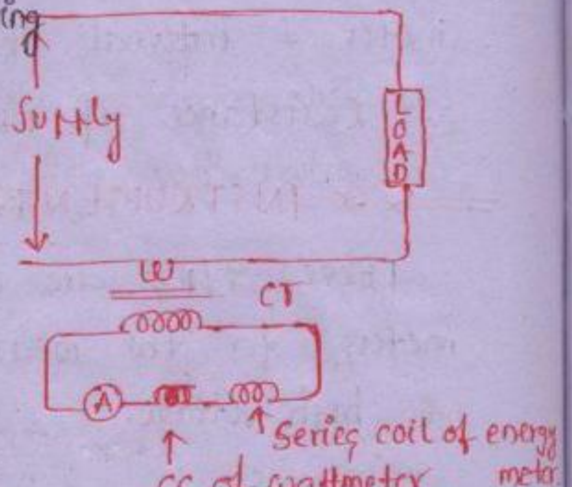
Burden:

load on T/F will be specified with the name of burden. It will be expressed in V-A.

CURRENT TRANSFORMER (CT):

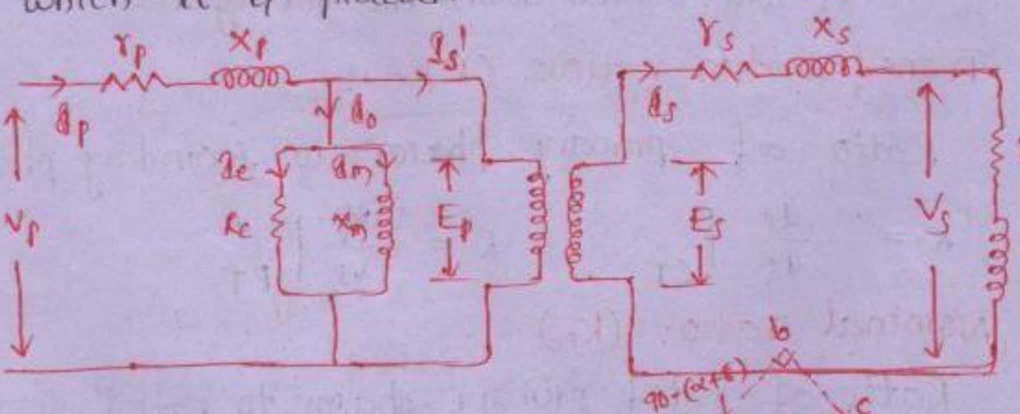
The secondary current rating CT is standardaized to 1A or 5A.

Usually 1A will be used for measurement and 5A for protection.



No. of turns on primary will be less (preferably 1 turn) at this condi this is known as bar primary CT.

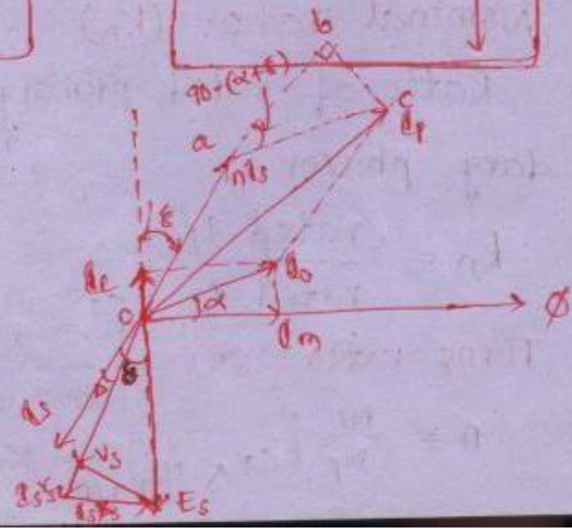
The p. wdg current in CT depends on ckt in which it is placed.



$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$I_p = \frac{N_s}{N_p} \cdot I_s$$

$$\rightarrow I_p = n \cdot I_s$$



From the phasor diagram, $I_p^2 = (ob)^2 + (bc)^2$

$$\Rightarrow I_p^2 = (oa + ab)^2 + bc^2$$

$$= \left(nI_s + I_0 \cos[90 - (\alpha+\delta)]\right)^2 + [I_0 \sin(90 - (\alpha+\delta))]^2$$

$$= (nI_s + I_0 \sin(\alpha+\delta))^2 + (I_0 \cos(\alpha+\delta))^2$$

$$= n^2 I_s^2 + 2 I_0 n I_s \cdot \sin(\alpha+\delta) + I_0^2$$

$$\simeq n^2 I_s^2 + 2 I_0 n I_s \sin(\alpha+\delta) + I_0^2 \sin^2(\alpha+\delta)$$

$$I_p \simeq nI_s + I_0 \sin(\alpha+\delta)$$

Transformation Ratio $(R) = \frac{I_p}{I_s}$

$$\Rightarrow R = n + \frac{I_0}{I_s} \cdot \sin(\alpha+\delta)$$

200 + (1.8)

198

$$\tan\theta = \frac{bc}{ob}$$

$$= \frac{I_0 \sin(90 - (\alpha+\delta))}{nI_s + I_0 \cos(90 - (\alpha+\delta))}$$

$$= \frac{I_0 \cos(\alpha+\delta)}{nI_s + I_0 \sin(\alpha+\delta)}$$

$$\tan\theta \simeq \frac{I_0 \cos(\alpha+\delta)}{nI_s}$$

$$\Rightarrow \theta \simeq \frac{I_0 \cos(\alpha+\delta)}{nI_s}$$

Ratio Error $= \frac{k_n - R}{R} \times 100$

phase angle error $(\theta) = \frac{I_0 \cos(\alpha+\delta)}{nI_s}$ rad

$$= \frac{180}{\pi}\left[\frac{I_0 \cos(\alpha+\delta)}{nI_s}\right] \text{deg}$$

200/1 → CT

$k_n = \frac{200}{1} = 200$

$N_p = 1$

$N_s = 200$ 198

$n = \frac{N_s}{N_p} = \frac{200}{198}$

In an uncompensated CT, turns ratio will be same as nominal ratio. Then transformation ratio will be more than nominal. resultant ratio error will always be -ve.

Turns Compensation :- (for only secondary)

$K_n = \frac{N_s}{N_p}$ To appr. equalling the transformation ratio to nominal ratio it is preferable to select no. of turns on secondary to be less. This is known as turns compensation.

(In primary, we can't change the turns b'coz it is in the series with [illegible].)

The cause of errors in CT is due to NL current of T/f. It consists of core loss comp. & magnetising component.

CT is also known as series T/f and its secondary is almost operated under S/c'ed condition.

CT secondary should not be open circucted while primary is energised. It may result into

(a). Generation of high o/c volt. across secondary terminals.

(b). Core may get saturate.

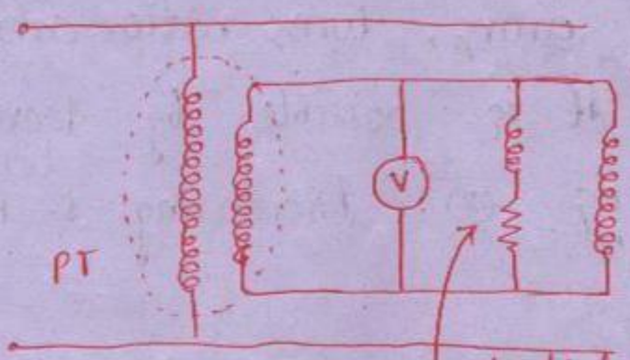
(c). Insulation of CT may get damage.

* 25/12/08 *

Potential Transformer: (PT):

PT scaled down volt. and electrically equal to s/ down T/f.

It is known as ||^{el} T/f. and secondary of PT almost operated under o/c condition.



The S. volt. rating of PT is standardised to 110V.

Transformation ratio $R = \frac{V_P}{V_S}$

$$= n + \frac{\frac{I_s}{n}\left[R_p \cos\Delta + X_p \sin\Delta\right] + I_c r_p + I_m x_p}{V_s}$$

$$\theta = \frac{\frac{I_s}{n}\left[X_p \cos\Delta - R_p \sin\Delta\right] + I_c x_p - I_m r_p}{n V_s} \text{ rad.}$$

R_p, X_p = equivalent resistance & reactance refer to primary.

r_p, x_p = resistance, reactance of primary wdg only.

$\Delta = \tan^{-1}\left(\frac{x_e}{r_e}\right)$

r_e, x_e = equivalent resistance, reactance coming on PT secondary.

for uncompensated PT turns ratio = nominal ratio then transformation will be more than nominal ratio.

Turns compensation :-

To reduce errors, turns ratio can be reduced (1). It is possible by decreasing p. no. of turns (2). Increasing s. no. of turns.

$n = \frac{N_s \uparrow}{N_p \downarrow}$

For the volt's above 100 kV; capacitive, potential T/F's [CVT's] are used for measurement purpose.

6.1 $f = 50\ Hz$

$N_p = 1$

$N_s = 200$

$r_e = 1\Omega$

$I_s = 5A$

$AT = 80$

$A = 10\ cm^2$

$V_s = I_s \cdot r_e = 5 \times 1 = 5V$

(Secondary volt.)

(a). $V_p = \frac{N_p}{N_s} \times V_s$

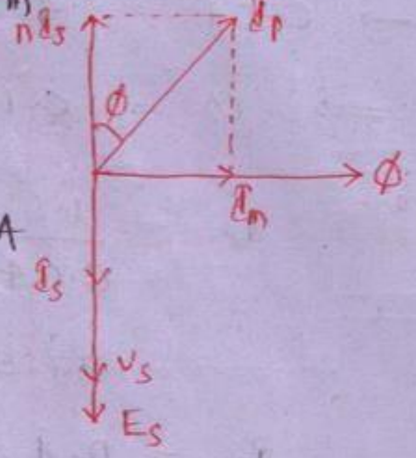
(p. volt.)

$= \frac{1}{200} \times 5 = 0.025V$

$V_p = 4.44\ B_m \cdot A\ f\ N_p$

$$0.025 = 4.44 \times 10 \times 10^{-4} \times B_m \times 50 \times 1$$

$$\Rightarrow B_m = 0.1125 \ wb/m^2$$



(b).

$$N_p \times I_m = 80$$

$$\rightarrow I_m = \frac{80}{1} = 80A$$

(magnetising current)

$$I_p = \sqrt{(nI_s)^2 + I_m^2}$$

$$= \sqrt{(200 \times 5)^2 + (80)^2}$$

$$= 1003.2 \ A.$$

Trans. ratio $R = \frac{I_p}{I_s} = \frac{1003.2}{5} = 200.64$

$$\tan\theta = \frac{I_m}{n.I_s} = \frac{80}{5 \times 200}$$

$$\Rightarrow \theta = 45.7°.$$

6.2.

$$k_n = \frac{1000}{5} = 200$$

$$f = 50 \ Hz$$

$$r_e = 1.6 \ \Omega$$

$$N_p = 1$$

Iron loss = 1.5 ω

$$V_s = I_s . r_e$$

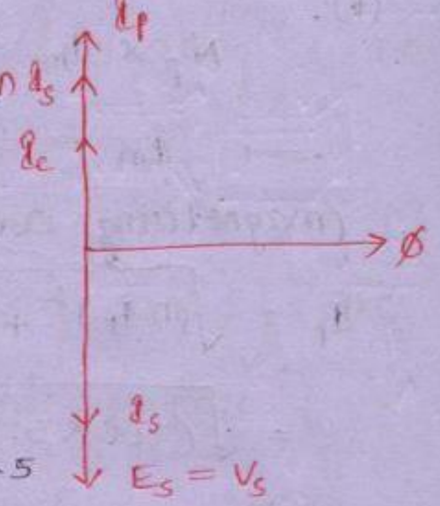
$$= 5 \times 1.6 = 8V$$

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} \Rightarrow V_p = \frac{1}{200} \times 8 = 0.04 \ V$$

$$V_p = 4.44\, \phi f N_p$$

$$\rightarrow 0.04 = 4.44 \times \phi \times 50 \times 1$$

$$\Rightarrow \phi = 0.18 \text{ mwb}$$



(b).

$$I_c = \frac{1.5}{0.04}$$

$$= 37.5 \text{ A}$$

$$I_p = n.I_s. + I_c$$

$$= (200 \times 5) + 37.5$$

$$= 1037.5 \text{ A}$$

$$R = \frac{I_p}{I_s} = \frac{1037.5}{5} = 207.5$$

$$\text{Ratio error} = \frac{k_n - R}{R} \times 100\%$$

$$= \frac{200 - 207.5}{207.5} \times 100\%$$

$$= -3.6\%$$

6.3. $N_p = 1$

$$k_n = \frac{1000}{5} = 200$$

$$r_e = 1\Omega$$

$$I_0 = 1\text{A}$$

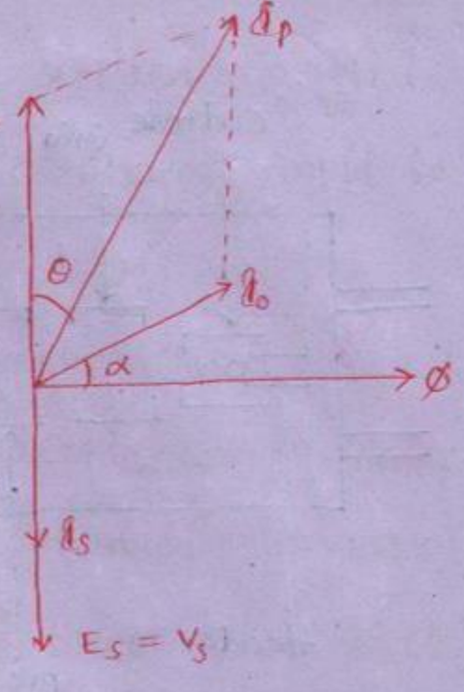
$$\text{NL } pf = 0.4$$

$$\alpha = 90 - \cos^{-1}(0.4)$$

$$= 23.57^\circ$$

$$R = n + \frac{I_0 \sin(\alpha + \delta)}{I_s}$$

$$= 200 + \frac{1 \times \sin(23.57 + 0)}{5}$$



$$R = 200.08$$

$$\theta = \frac{I_0 \cos(\alpha + \delta)}{n \cdot I_s} \cdot \frac{180}{\pi} \text{ deg.}$$

$$= \frac{1 \times \cos(0 + 23.57)}{200 \times 5} \cdot \frac{180}{\pi}$$

$$= 0.052^\circ$$

$$\text{Ratio Error} = \frac{K_n - R}{R} \times 100\%$$

$$= \frac{200 - 200.08}{200.08} \times 100\%$$

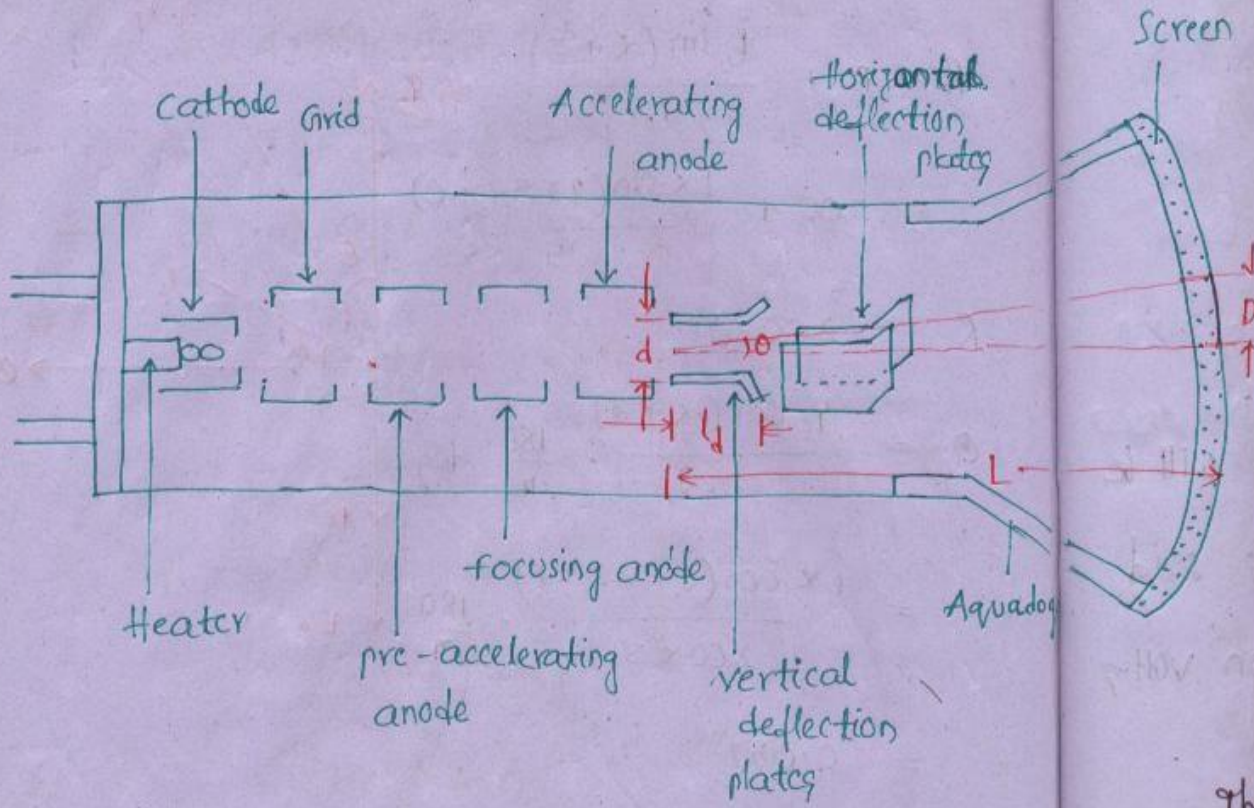
$$= -0.04\%$$

CRO :

CRO is a very useful laboratory instr. used for display, measurement & analysis of wave form.

CRO can be used for higher freq. [Duddell's Oscilloscope]

The main element of CRO is CRT.



The main parts of CRT are–

(a). e^- gun assembly

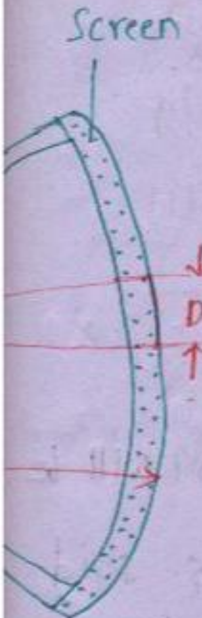
(b). deflection plate assembly

(c). fluoroscent screen

(d). Glass envolope.

e^- gun assembly produces sharp focused beam of e^-s which are accelerated to high velocety with the help of focusing anode, accelerating anodes etc.

focused beam of e^-s straike the screen with suffecient energy to produce luminous spot on screen.



The potentials applied to grid and subsequent electrodes would be in the following range.

grid → −ve potential

pre accelerating, accelerating anodes ⟶ High +ve potential (1500 V).

focusing anode → lower adjustable +ve voltage [upto 500 V].

The e^- beam focused on the screen in two methods.

(1). Electro static focusing : (used for CRO applications].

(2). Electro magnetic focusing : [for TV applications].

e^- beam will be deflected by the potentials applied to deflection plates. Electro static means of deflection is employed for the deflection of $\bar{e}$ beam

$$\text{Deflection } (D) = \frac{L \cdot l_d}{2d \times E_a} \times E_d .$$

$$\text{Deflection sensitivity } (S) = \frac{D}{E_d}$$

$$\Rightarrow \quad S = \frac{L}{2d} \cdot \frac{L_d}{E_a}$$

Deflection factor $= \frac{1}{S}$.

E_a = accelerating anode volt.

E_d = Deflection plate volt.

If E_a is more then the beam will be highly accelerated and then it is said to be hard beam. High deflection volts required to deflect hard beam.

Aquadog :-

Whenever $\bar{e}$ beam straikes fluorosent screen it produces secondary emission $\bar{e}$s. So collect these $\bar{e}$s Aquadog is employed. It is an homogeneous soln. of graphite coated around glass envolope.

The $\bar{e}$ beam needs acceleration after deflection if signals of more than 10 MHz are to be displayed. Post deflection Accelarators are used for this purpose.

Display of unknown Signal:

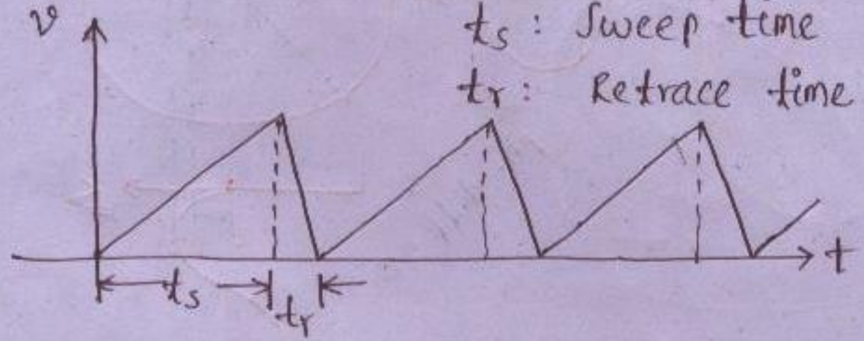
For the display of any unknown

signal the volt-s applied to deflection plates are as follows.

(1). vertical deflection plates (Y-plates) — unknown signals.

(2). Horizontal deflection plates - sweep signals.

In general saw tooth wave is used as a sweep signal.



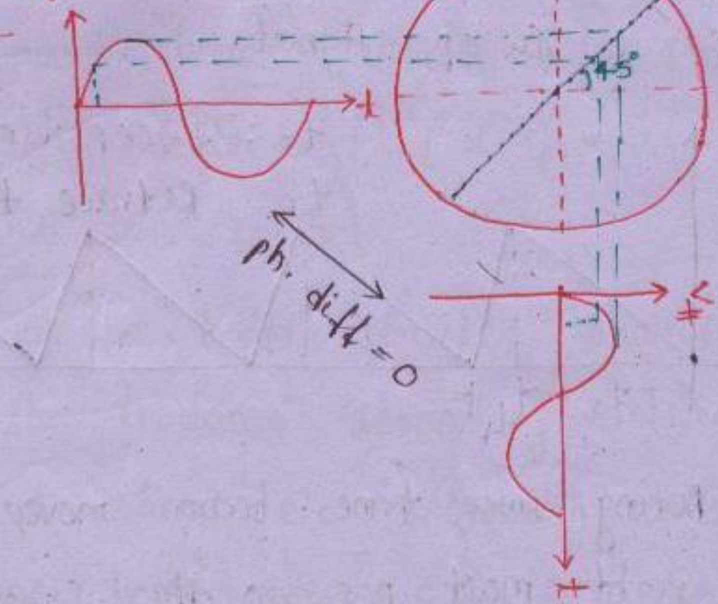
During sweep time beam moves from left to right most pos. on the screen.

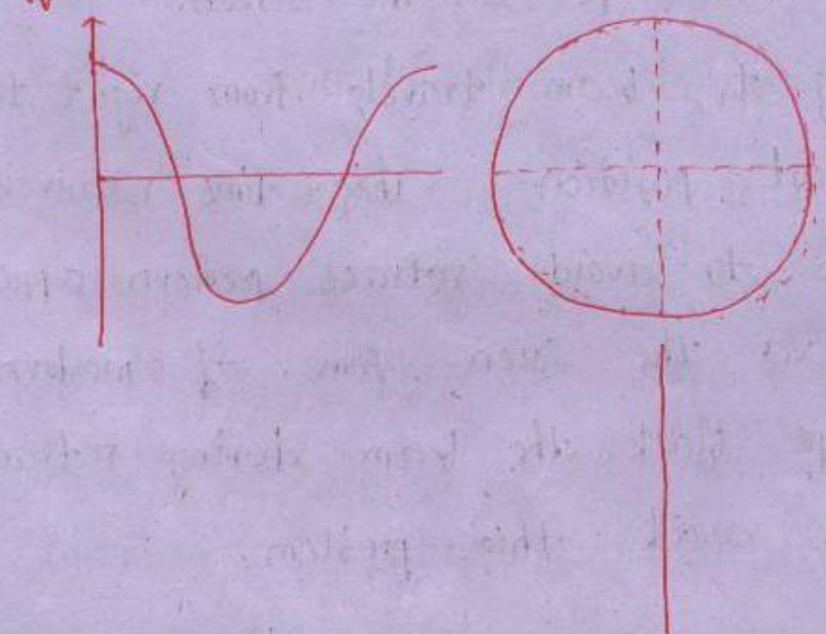
During t_r, beam travels from right to left most position. This time required is v.v. less to avoid retrace pattern appearance over the screen. some of modern oscilloscope block the beam during retrace time to avoid this problem.

Measurement of phase & freq. :

In the measurement of ph. & freq. sinusoidal volts will be applied to both horizontal & vertical deflection plates. Then resultant pattern appearance on screen known as Lissajous patterns.

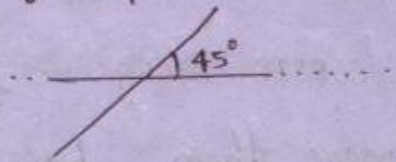
for phase :

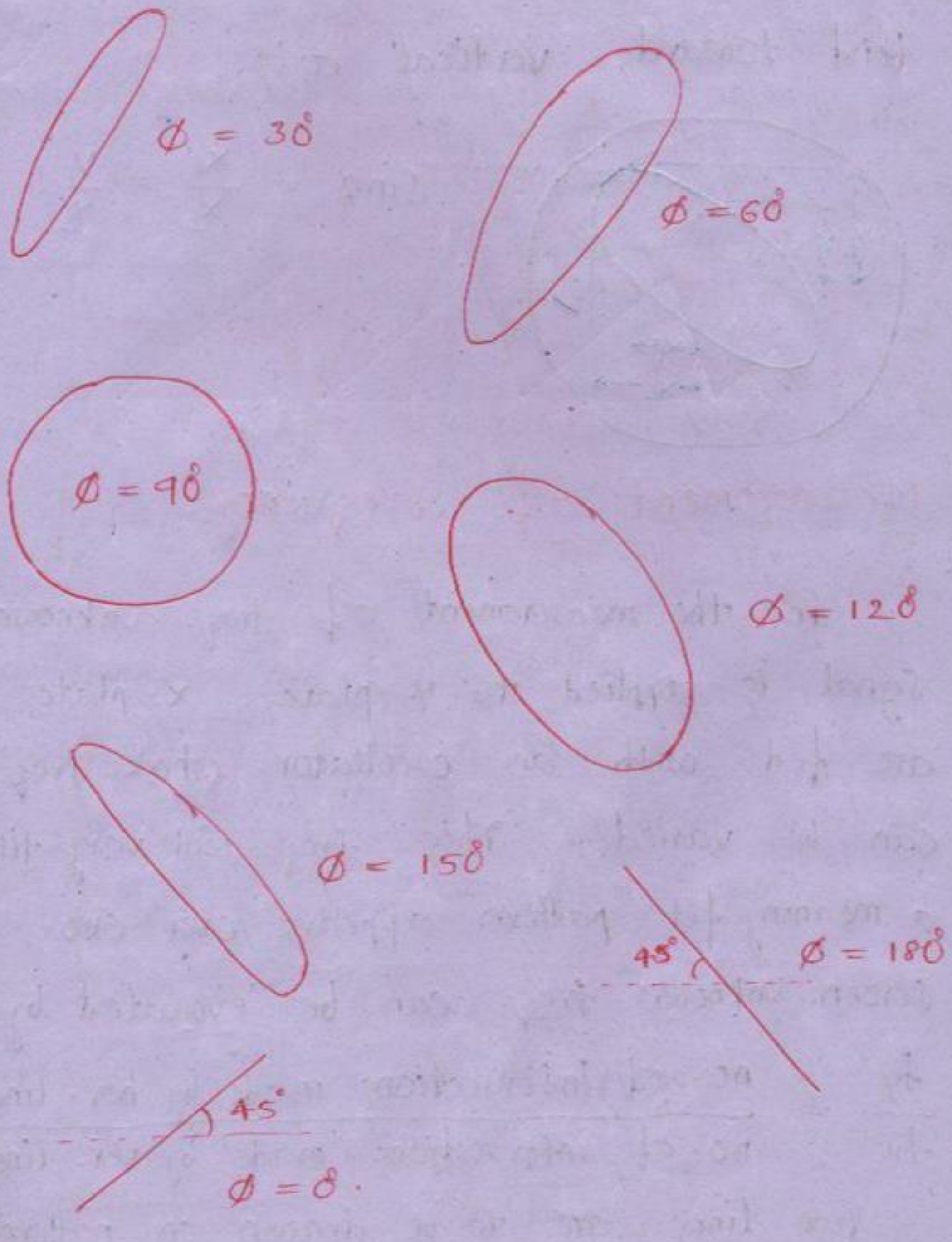




whenever equal amount of voltages are applied to both x-plates & y-plates, the shape of pattern for different ph. angles are as follows.

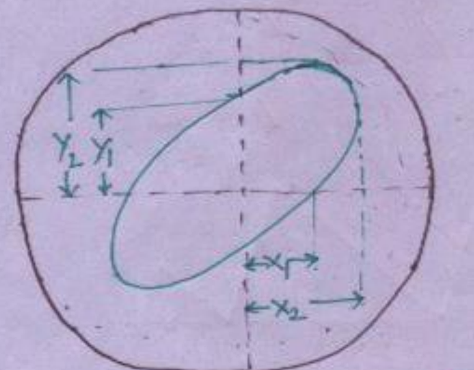
for $\phi = 0$ $\longrightarrow$





when the mag. of hor. plate volt more than ver. plate volt. then shape of pattern remain same but it bends towards hor. axis.

when mag. of vertical plate volt is more than hor. plate volt. then pattern bend towards vertical axis.



$$\sin\phi = \frac{x_1}{x_2} = \frac{Y_1}{Y_2}$$

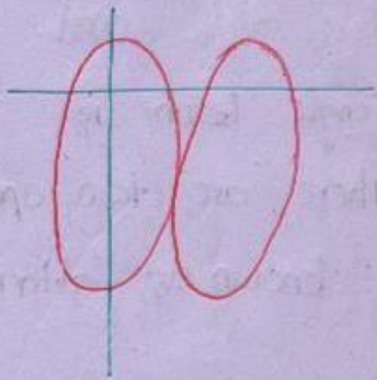
Measurement of frequency:

In the measurement of freq. unknown signal is applied to y-plates. x-plates are fed with an oscillator whose freq. can be varied. This freq. will vary till a meaningful pattern appears over CRO screen. Unknown freq. can be evaluated by

$$\frac{f_y}{f_x} = \frac{\text{no. of intersections made by hor. line}}{\text{no. of intersections made by ver. line}}$$

Two lines are to be drawn on pattern

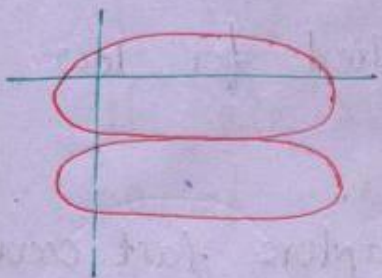
one is hor. & another is ver. These lines are to be drawn in such a way they should not pass through any intersection of curve and pass through whole curve.



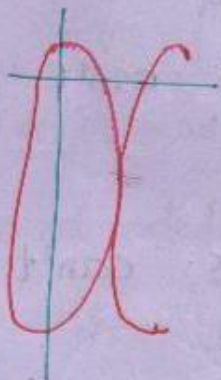
$f_x = 50\ Hz$

$\frac{f_y}{f_x} = \frac{4}{2}$

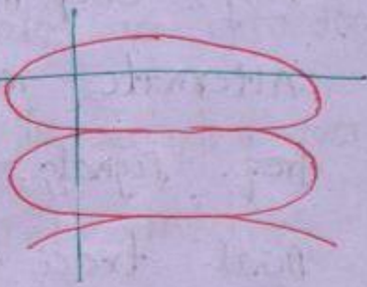
$\therefore f_y = \frac{4}{2} \times 50 = 100\ Hz.$



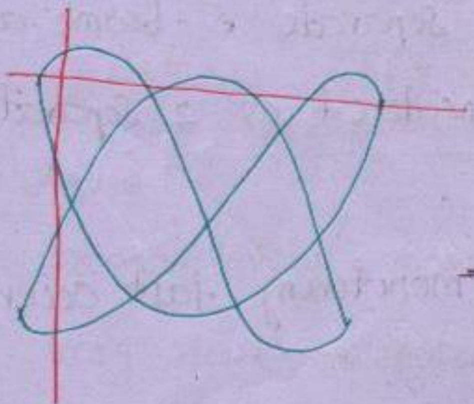
$\frac{f_y}{f_x} = \frac{2}{4}$



$\frac{f_y}{f_x} = \frac{3}{2}$



$\frac{f_y}{f_x} = \frac{2}{5}.$



$\frac{f_y}{f_x} = \frac{6}{4}$

$\longrightarrow f_y = \frac{6}{4} \times 50$

$= 75\ Hz.$

$\longrightarrow$ 12 spikes.

so $\frac{f_y}{f_x} = 12$

Special type Oscillo scopes :

Dual i/p oscilloscopes :

2 types : (a). Dual trace

(b). Dual beam

Dual trace O.S :

In this CRO, a single beam is splitted into 2 traces. There are two operating modes for operating known as alternating, chop mode.

Alternate mode can't applied for low freq. signals.

Dual trace O.S. can't capture fast occuring events.

Dual beam O.S :

It has got two separate ē beams and therefore can be considered as 2 separately vertical channels.

It is useful for monitoring fast occuring events also.

Storage O.S :

It is capable of retaining image on screen for longer time.

It is suitable for capture and storage of non - repetative waveforms like transients.

Storage mesh may be used to retain image for longer time. Magnisium flouride is used in making of storage mesh.

Sampling O.S. :

It is suitable to capture and display of very high freq. signals ie upto 300MHz

The i/p wave form will be sampled over no. of cycles at different intervals of time w.r.t its origin. Based on sampled values the wave form can be reconstructed over CRO screen.

DIGITAL VOLTMETERS [DVM] :

Adv.s :

1. comparitively more accurate meters
2. very fast response
3. i/p resistance of DVM is very high hence loading effect on unknown signal is negligible.

3. I/p volt. range can be varied to reduce errors in the measurement.

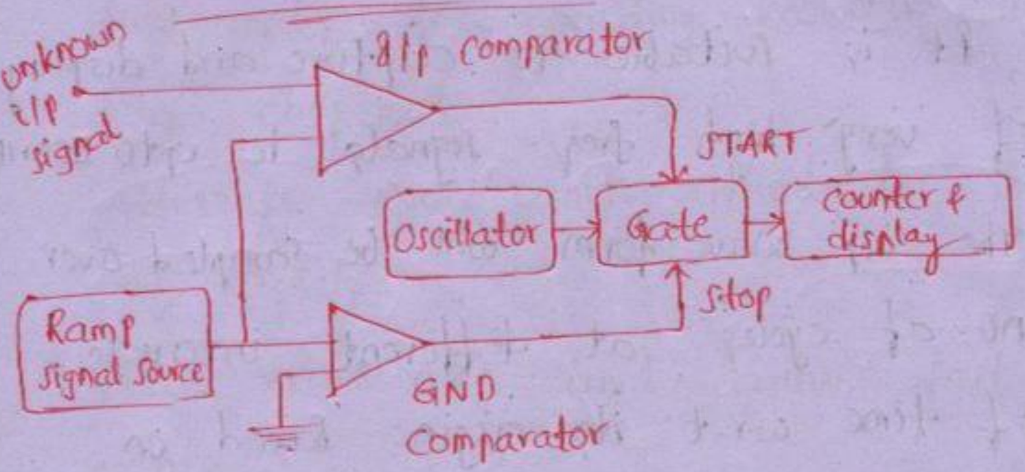
DVM's are of 3 types.

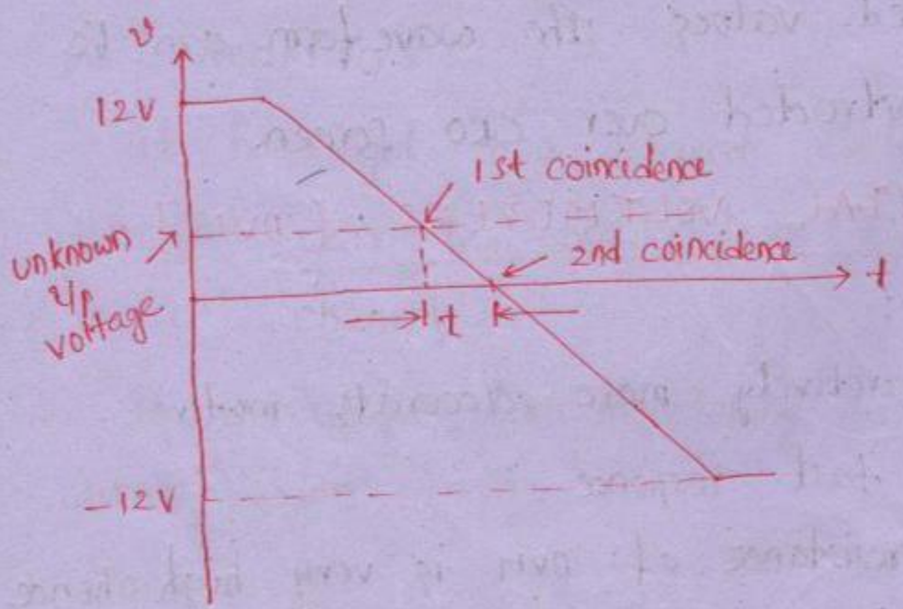
(a). ramp type DVM

(b). Dual slope integrating type DVM.

(c). Integrating type DVM.

RAMP TYPE DVM:





Operating principle is to measure the time that a linear ramp takes time to change from unknown i/p volt. level to ground

level. [volt to time conversion].
I/p comparator identifies 1st coincidence and issues start pulse. Ground compa. identifies 2nd coincidence and issue stop pulse to gate.

Counter counts CP's from START to STOP pulses of gate. Unknown volt. is evoluated by multiplying time with slope of ramp signal.

Large errors are possible when noise is superimposed on i/p signal.